# Neuroinformatics

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Week 10: Cortical maps and competitive population coding
(textbook chapter 7)

# Outline

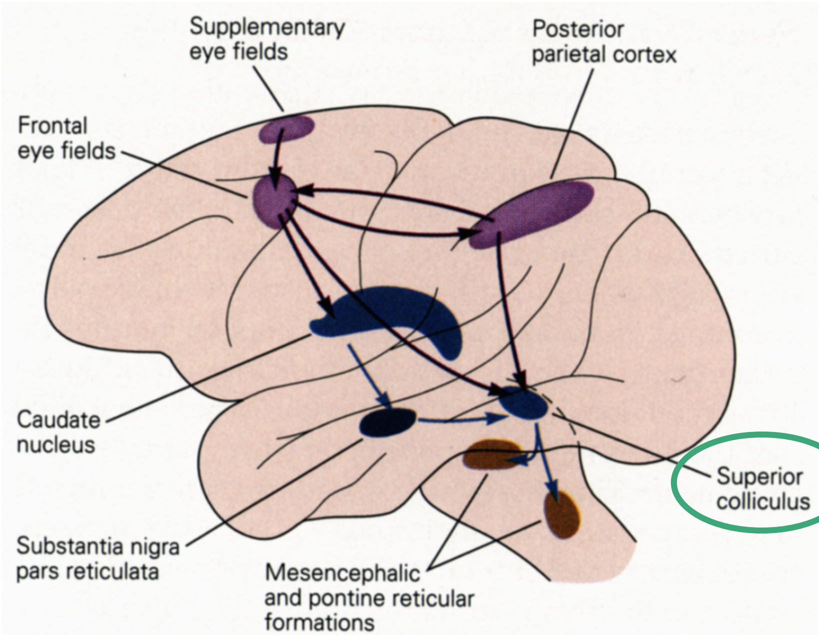
Topographic maps

Self-organizing maps

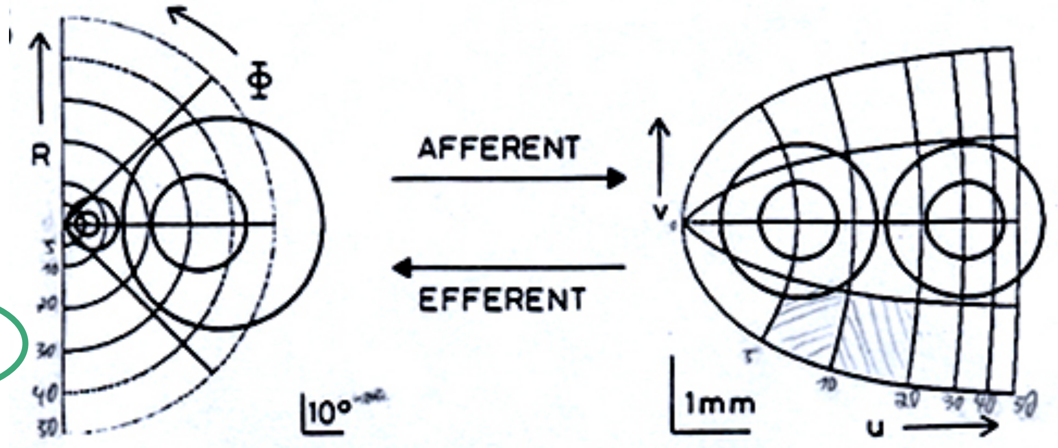
- Willshaw & von der Malsburg
- Kohonen
- Dynamic neural Field

# Topographic mapping

Topographic map: neighborhood relation is preserved (but scaling is allowed!)

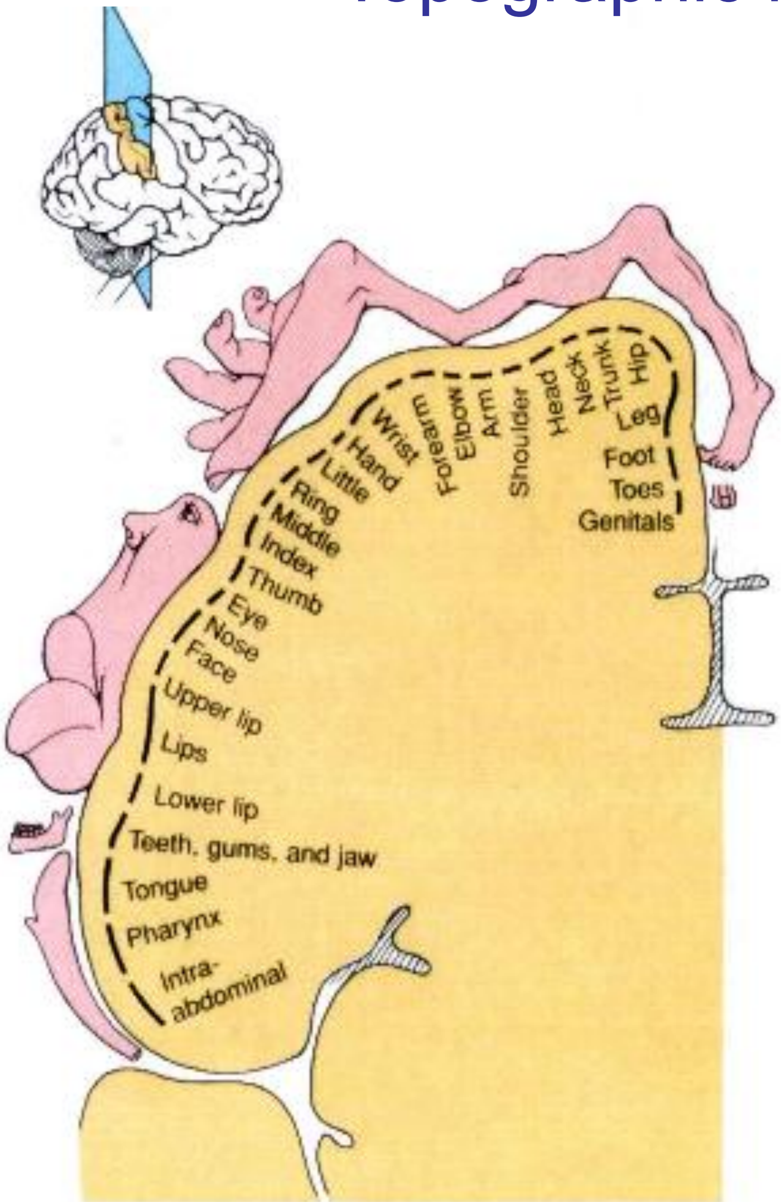



Pathway involved in generating fast eye movements (saccades)

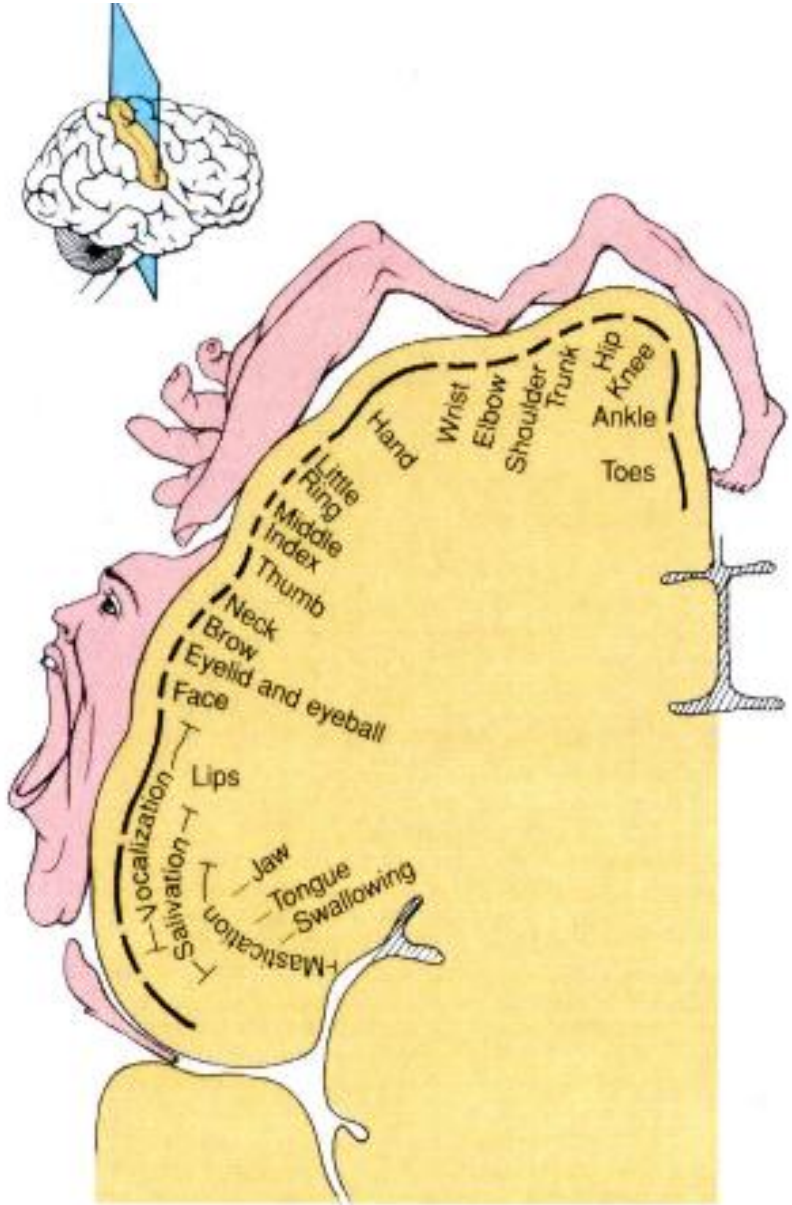



Mapping of objects in the external world (left) to the superior colliculus (right). R: angular eccentricity from fovea. Phi: angle

# Topographic map: Homunculus



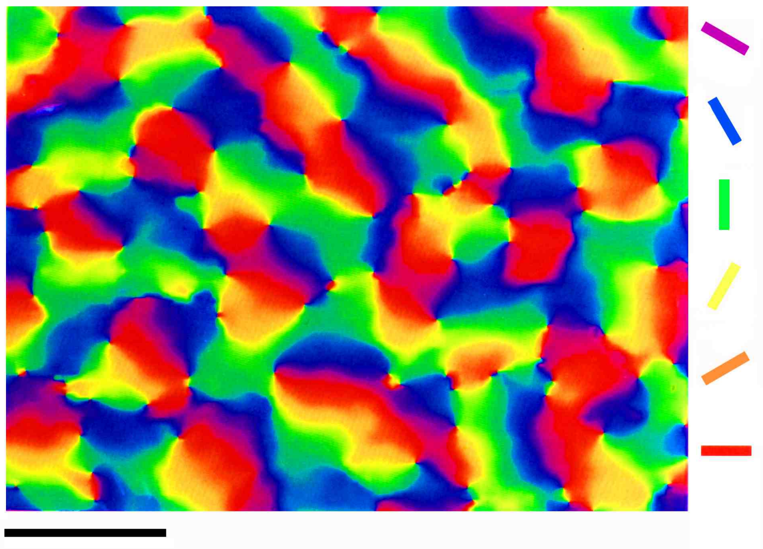

(a) Somatosensory cortex in right cerebral hemisphere




(b) Motor cortex in right cerebral hemisphere

# Topographic map: other examples

**Orientation map**

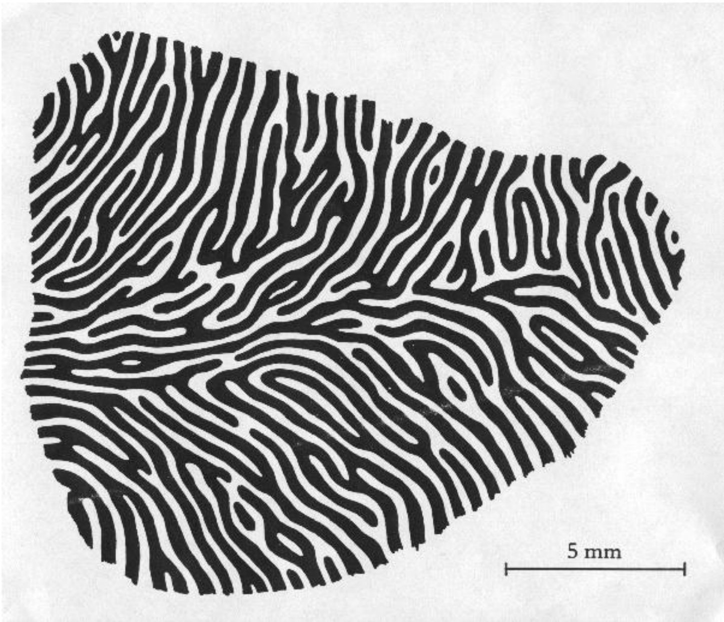


(http://www.scholarpedia.org/article/Visual_map)

•Hubel&Wiesel (1962, 1974): orientation selectivity and its locally continuity characteristic

•Swindale (1982),Blasdel&Salama(1986), Swindale et al.(1987): 2D map

**Ocular Dominance Columns**




Reconstruction of the ocular dominance columns in area 17 of the right Hemisphere of a monkey (tangential section)

# Topographic mapping?

Topographic map: neighborhood relation is preserved

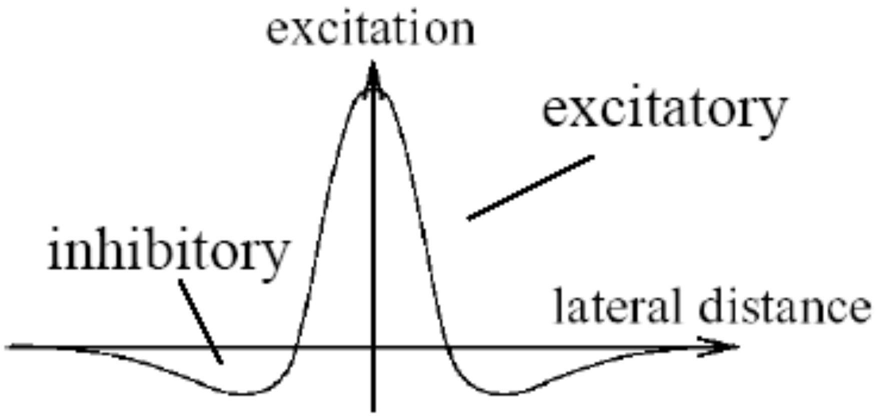
| Original map | Topographic map? | |
|---|---|---|
| | 3 2 1<br>6 5 4<br>9 8 7 | Yes (flipping, here along the x-axis) |
| 1 2 3<br>4 5 6<br>7 8 9 | 3 2 1<br>6 5 4<br>9 8 7 | Yes (scaling, here logarithmic) |
| | 1 2 9<br>4 5 6<br>7 8 3 | No (neighborhood relation destroyed) |

# How to generate a topographic map?

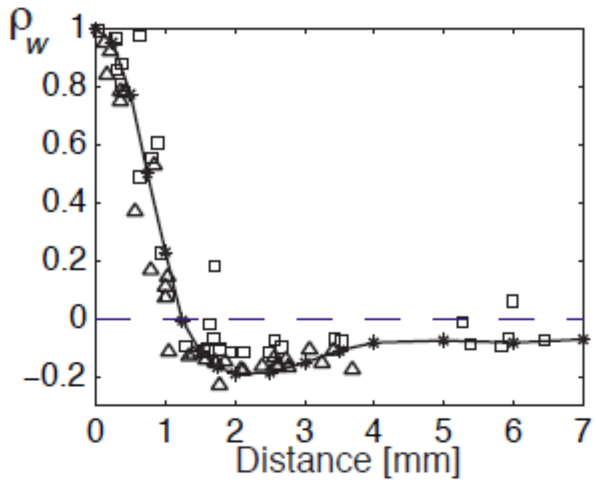
**Genetic encoding:**
The target for each projecting neuron is encoded by cell labeling or chemical gradients are used

**Alternative: self-organizing maps (SOM) using neural activity**
No encoding in the DNA necessary!

# A principle of SOM: cooperation and competition




Cooperation: Short-range excitation
Competition: long-range inhibition
(note: local inhibition)



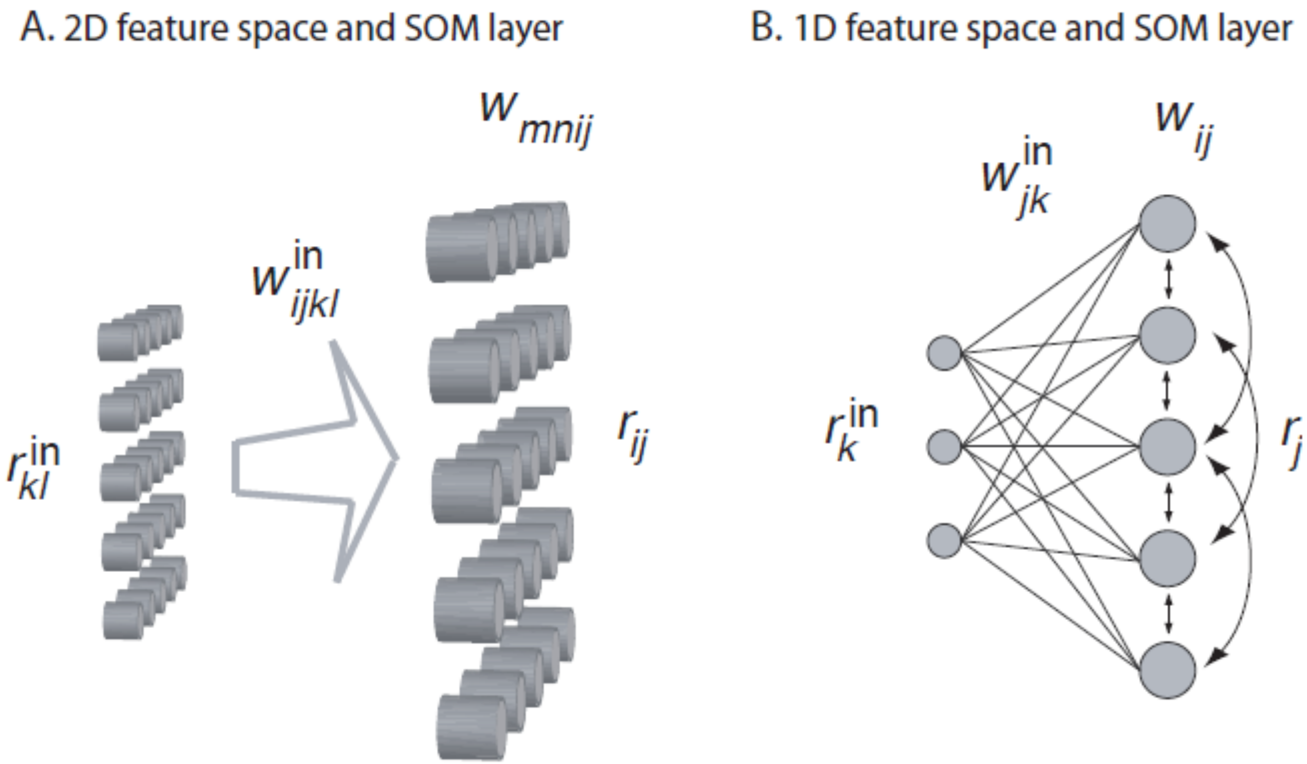

Interaction strength from cell recordings in superior colliculus (Trappenberg et al., 2001)

# Self-organizing maps (SOMs)

David Willshaw
Edinburgh Univ., UK

Willshaw-von der Malsburg model

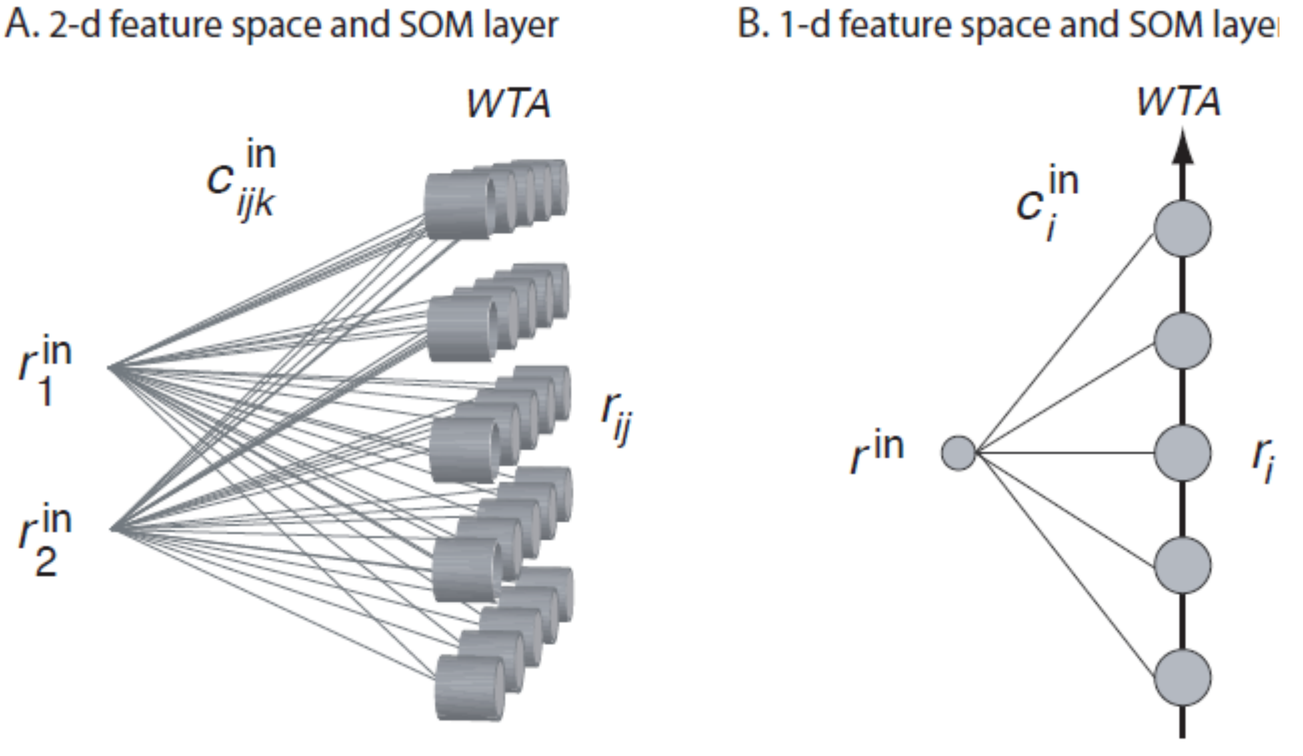
Christoph von der Malsburg
Bochum Univ. (now at FIAS, Frankfurt, Germany)



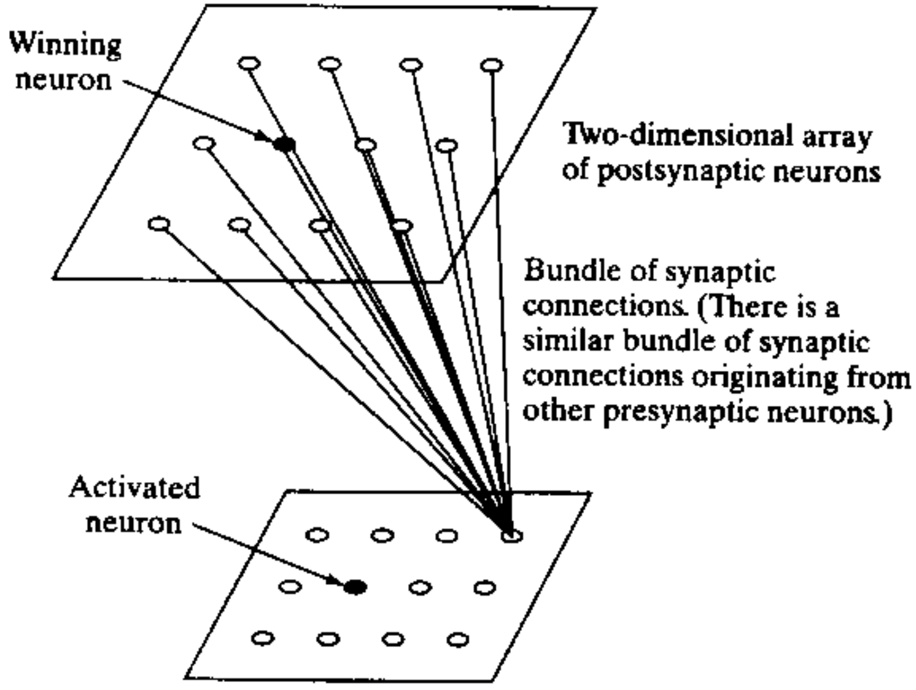
# Shortcut (no lateral connectivity)

Kohonen model

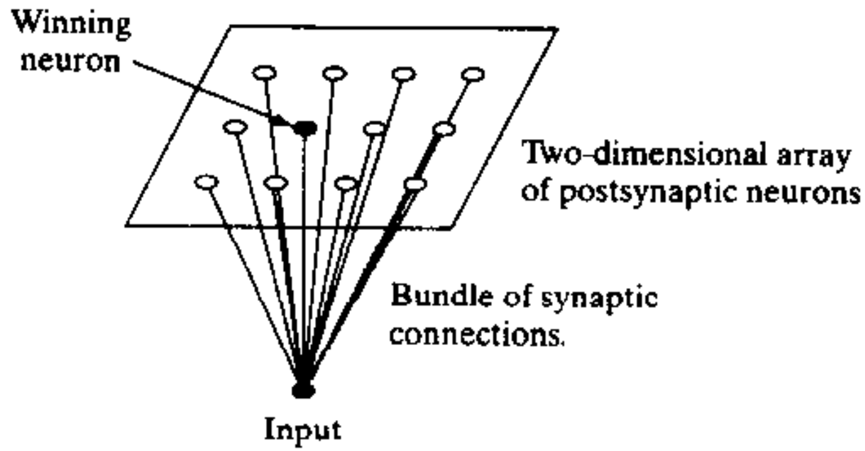
Teuvo Kohonen
Academy of Finland



# Two approaches for SOMs




(a) Willshaw-von der Malsburg's model




(b) Kohonen model

Developed for a retinotopic map
Input space is already topographic (retina)
Lateral connectivity captures C&C
The winning neuron occurs through neural dynamics
Can be both global and local competition

Input space is a continuous value
No lateral connectivity or neural dynamics
First find winning neuron (competition)
Then, learning of this neuron affects the neighbors (cooperation)
Global competition (no other possibility)

# Network equations

**Update rule of (recurrent) cortical network:**

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + \frac{1}{M}\sum_k w_{ik}^{\mathrm{in}} r_k^{\mathrm{in}}(t)$$

**Activation function:** $r_j(t) = \frac{1}{1+e^{\beta(u_j(t)-\alpha)}}$.

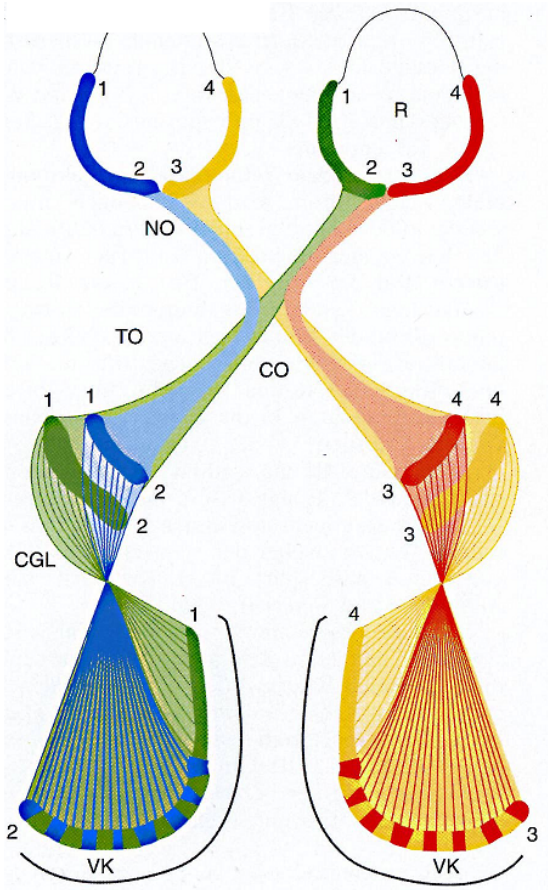
**Lateral weight matrix:** $w_{ij} \propto r_i r_j$

$$= A_{\mathrm{w}} \left( \mathrm{e}^{-((i-j)*\Delta x)^2/2\sigma^2} - C \right)$$

**Input weight matrix:** $w_{ij}^{\mathrm{in}} \propto r_i r_j^{\mathrm{in}}$

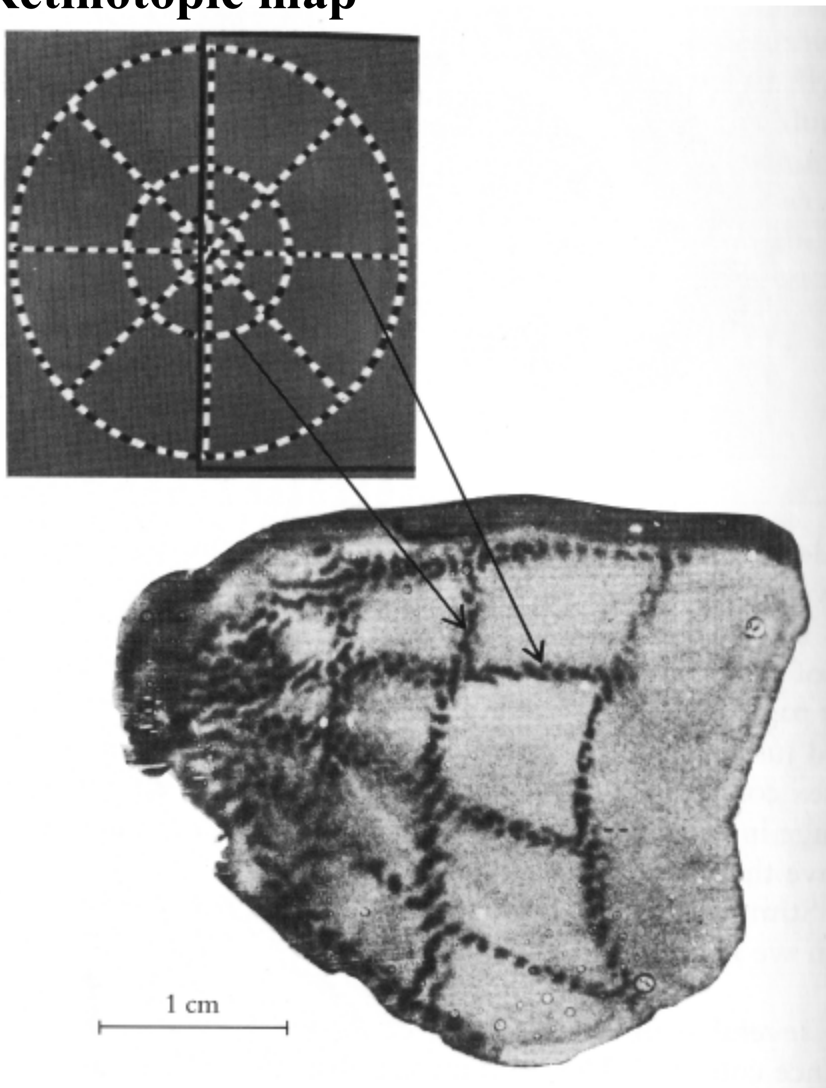
Stephen Grossberg
Boston Univ. USA

# Visual Pathway and Retinotopic map




R: retina
NO: nervus opticus
CO: chiasma opticum
TO: tractus opticus
CGL: corpus geniculatum lateralis
VK: primary visual cortex

**Retinotopic map**




(Tootel, 1983)

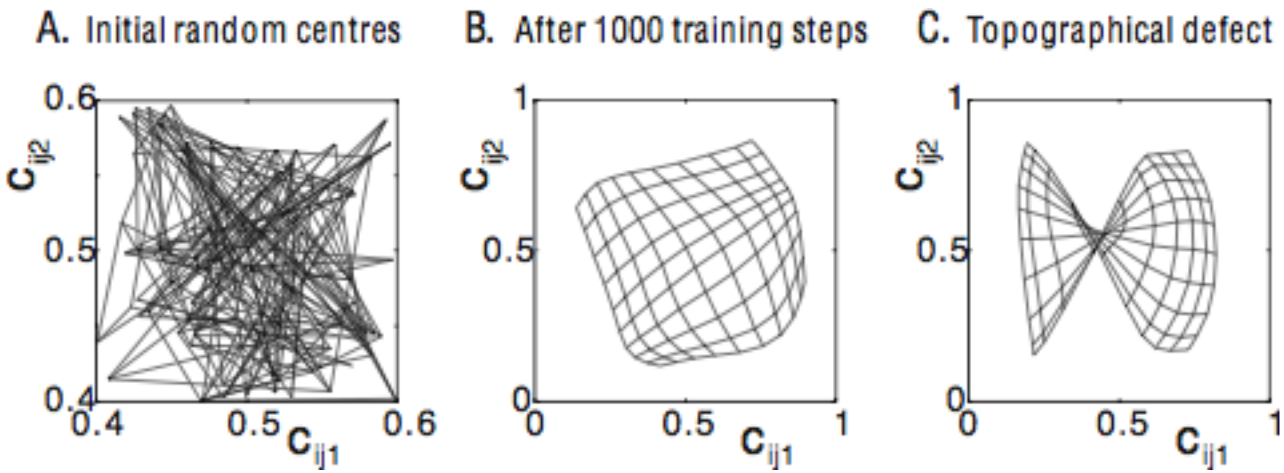
# som.m

```
%% Two dimensional self-organizing feature map al la Kohonen
 clear; nn=10; lambda=0.2; sig=2; sig2=1/(2*sig^2);
 [X,Y]=meshgrid(1:nn,1:nn); ntrial=0;

 % Initial centres of prefered features:
 c1=0.5-.1*(2*rand(nn)-1);
 c2=0.5-.1*(2*rand(nn)-1);

%% training session
 while(true)
    if(mod(ntrial,100)==0) % Plot grid of feature centres
        clf; hold on; axis square; axis([0 1 0 1]);
        plot(c1,c2,'k'); plot(c1',c2','k');
        tstring=[int2str(ntrial) ' examples']; title(tstring);
        waitforbuttonpress;
    end
    r_in=[rand;rand];
    r=exp(-(c1-r_in(1)).^2-(c2-r_in(2)).^2);
    [rmax,x_winner]=max(max(r)); [rmax,y_winner]=max(max(r'));
    r=exp(-((X-x_winner).^2+(Y-y_winner).^2)*sig2);
    c1=c1+lambda*r.*(r_in(1)-c1);
    c2=c2+lambda*r.*(r_in(2)-c2);
    ntrial=ntrial+1;
 end
```

# SOM simulations



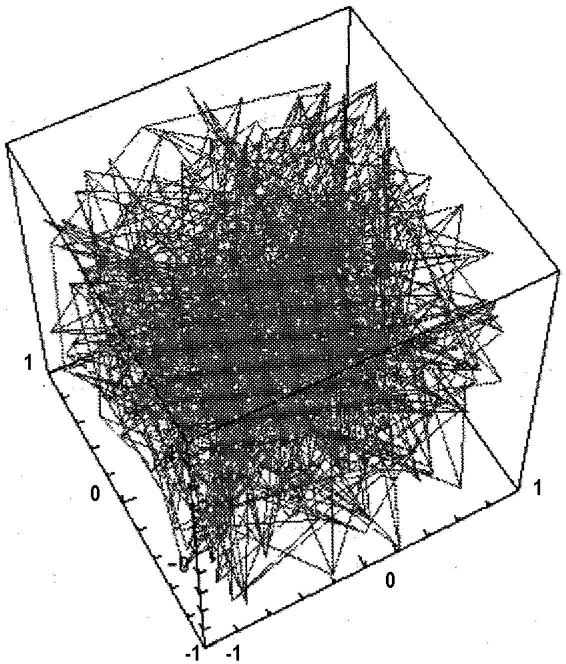

How to solve these defects?

(Similar to Simulated Annealing)

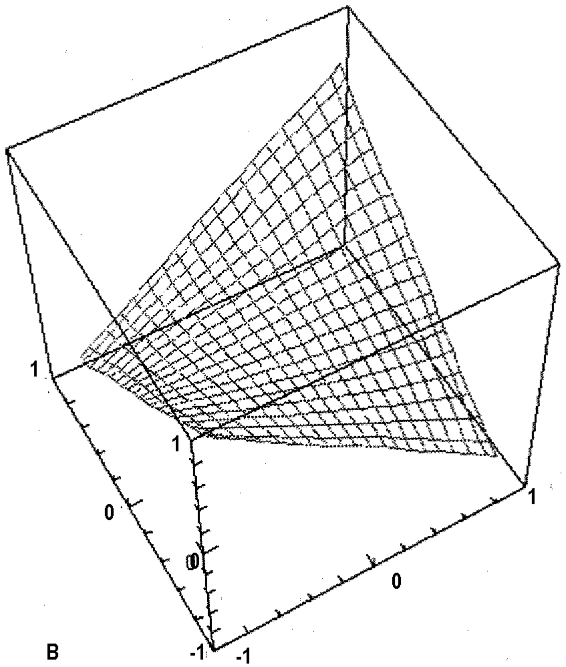
Two-phases: ordering and fine-tuning through decrease of extent

# Dimensionality Reduction

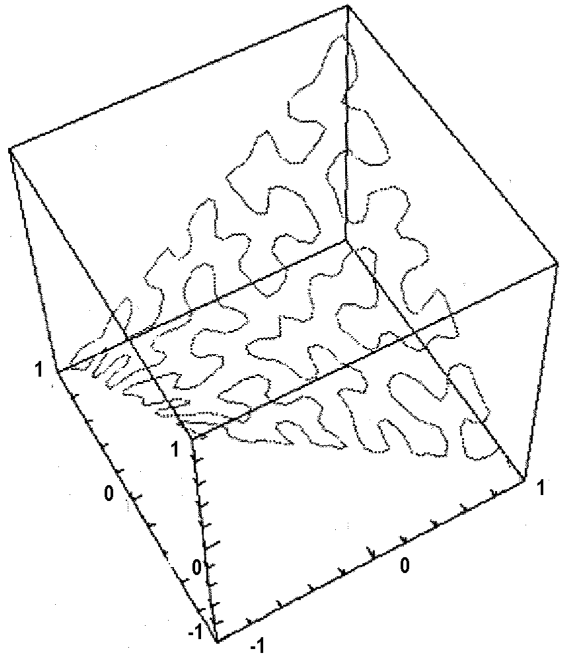
Input stimulus are random samples (uniformly distributed) on the surface z=xy



A



B



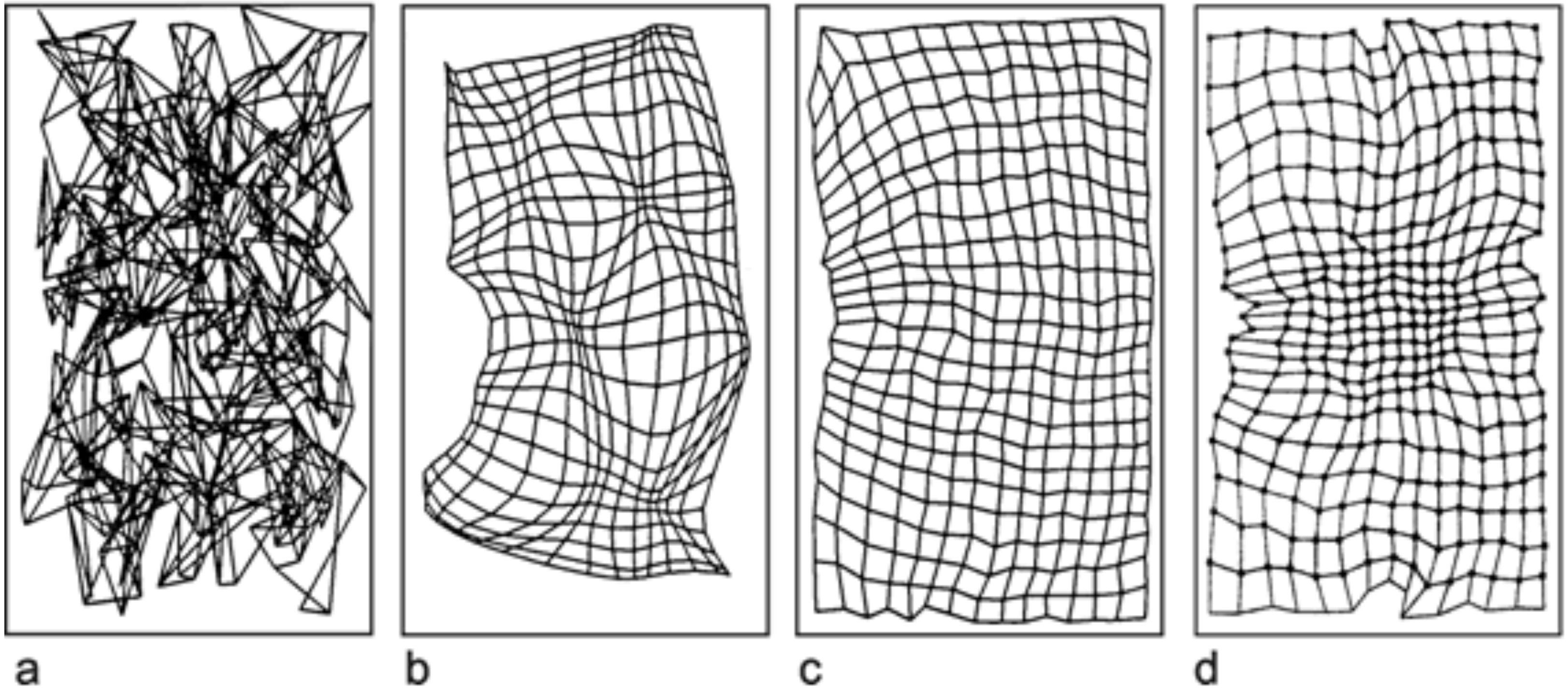
C

Initial state

2D map
(surface=2D)

1D map
(2D => 1D)

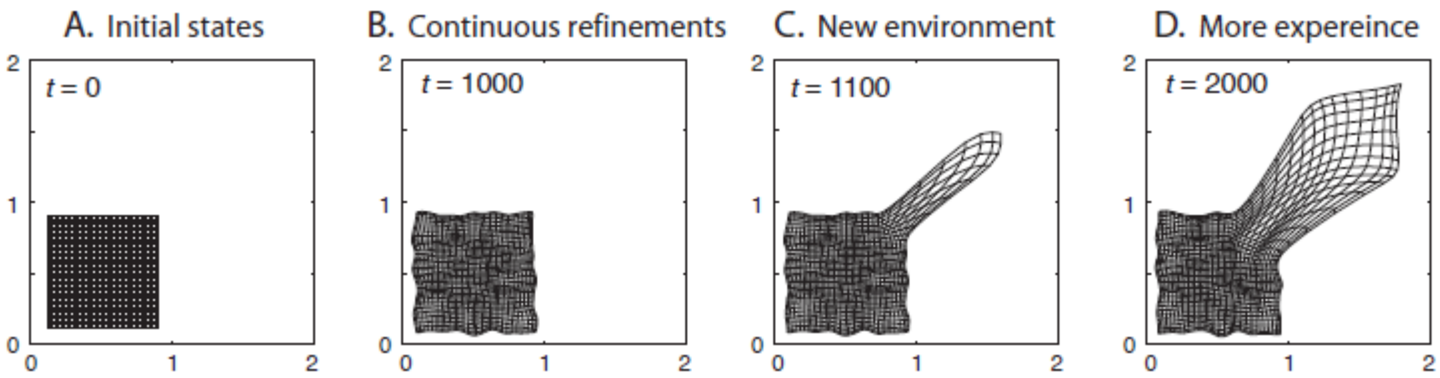
# What does a topographic map capture?




Initial state | On training | Trained | Trained with more samples on the center

Kohonen maps capture **stimulus (sample) density**.
More samples lead more units and consequently the map can distinguish more sensitively on those area => **receptive fields**!

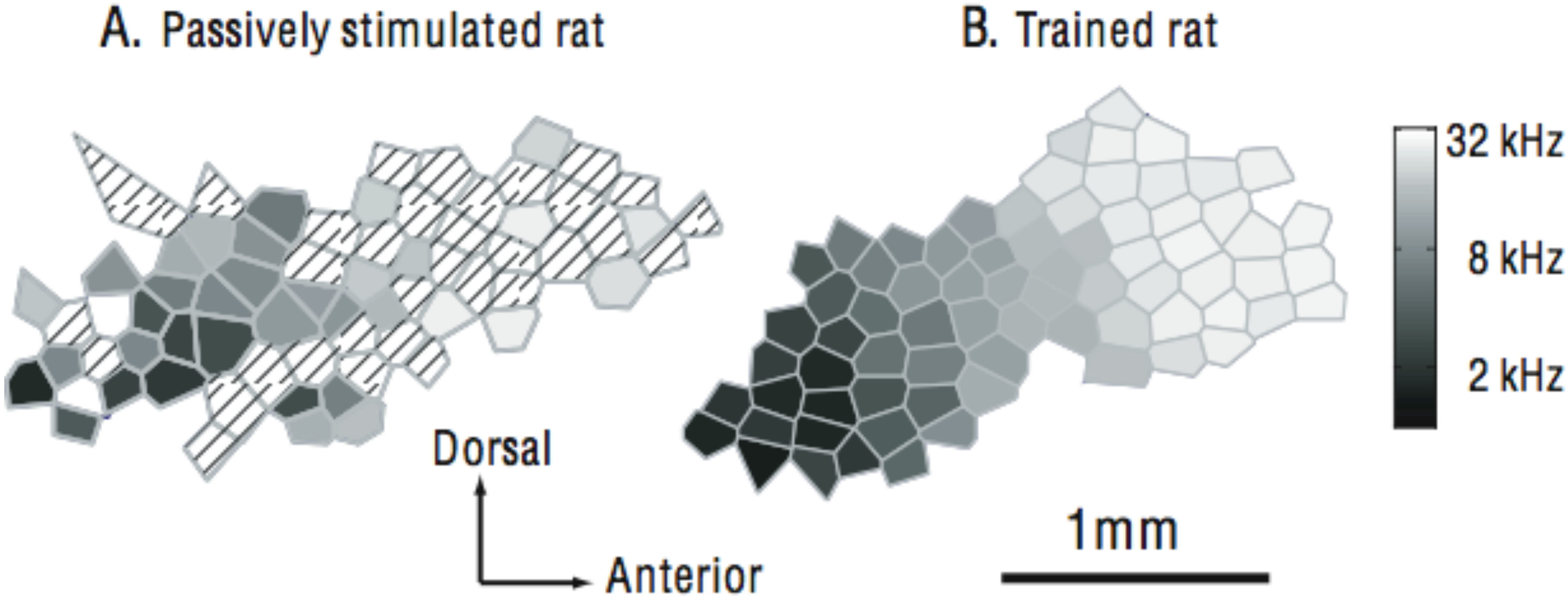
# Ongoing Refinements of cortical maps




New stimulus density is captured by a Kohonen map even after its map is settled.

# Zhou and Merzenich, PNAS 2007

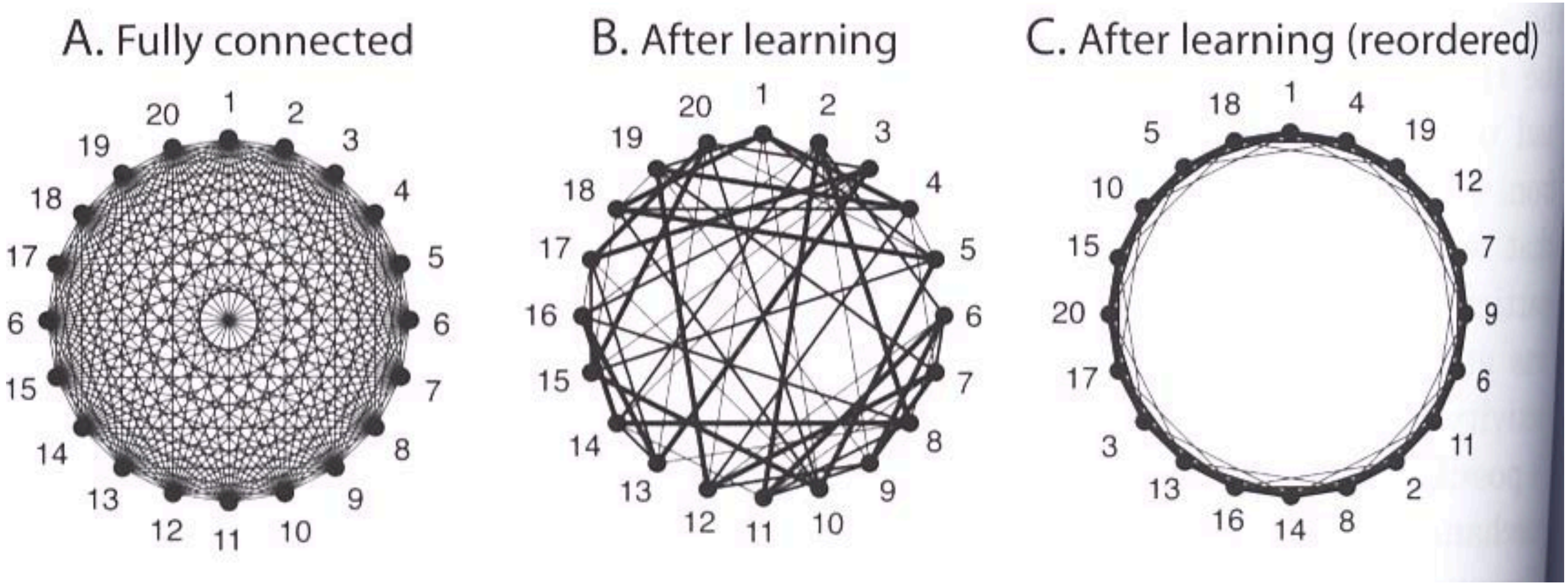
**Refinements of Frequency map in A1**




Raised in noisy environment

After training of a frequency discrimination task

# SOM and a network structure



# Dynamic Neural Field (DNF) Theory

**Field dynamics:**

$$\tau \frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} = -\mathbf{u}(\mathbf{x}, t) + \int_{\mathbf{y}} \mathbf{w}(\mathbf{x}, \mathbf{y}) \mathbf{r}(\mathbf{y}, t) \mathrm{d}\mathbf{y} + I^{\mathrm{ext}}(\mathbf{x}, t)$$

$$\mathbf{r}(\mathbf{x}, t) = g(\mathbf{u}(\mathbf{x}, t)),$$

Continuous version of equations above with discretization:

$$x \to i\Delta x \text{ and } \int \mathrm{d}x \to \Delta x \sum$$

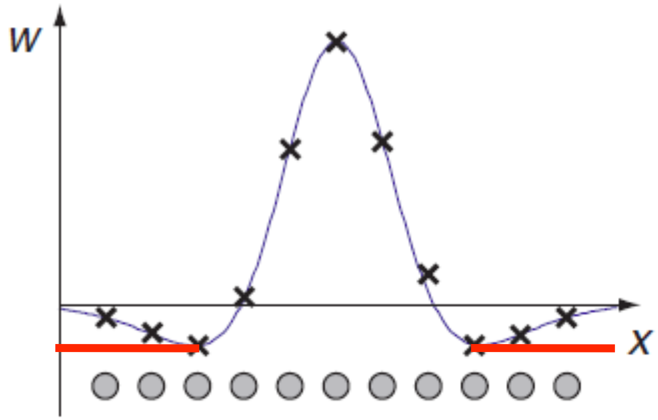
Distinct locations -> continuous locations (fields)

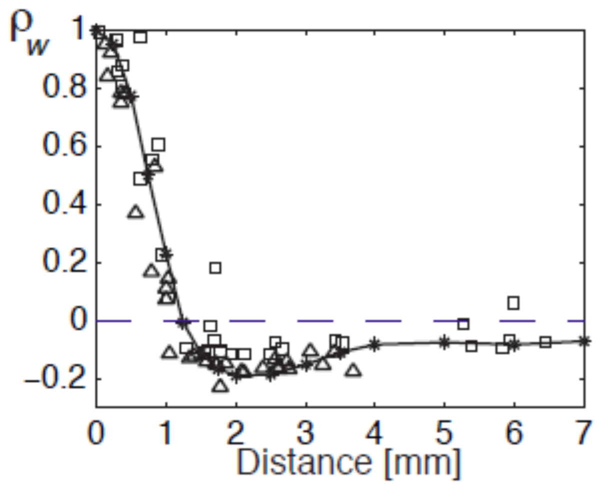
Very similar to Willshaw & von der Malsburg’s model.
Difference is no presynaptic layer (Instead, direct external inputs)

# The center-surround interaction (weight) kernel

$$\mathbf{w}^{\mathrm{E}}(|x-y|) = A_{\mathrm{w}} e^{-(x-y)^2/4\sigma_r{}^2} \; - A_w C$$

Can be learned from Gaussian response curves of individual nodes




Black solid line: a Mexican hat activation pattern (in 3D, local competition)

can be obtained with subtraction of two Gaussians.

matched with physiological data (right, Trappenberg et al., 2001)

Red Solid line: Gaussian with negative bias (global competition)

# dnf.m

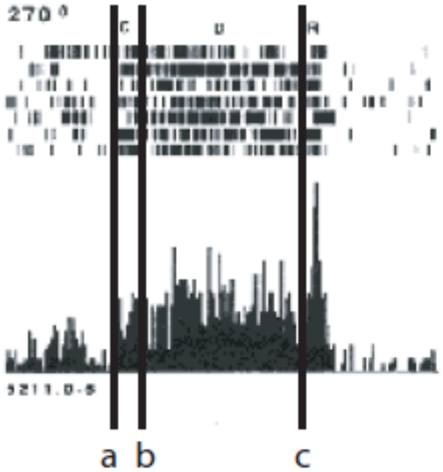
```
%% Dynamic Neural Field Model (1D)
 clear; clf; hold on;
 nn = 100; dx=2*pi/nn; sig = 2*pi/10; C=0.5;

%% Training weight matrix
 for loc=1:nn;
     i=(1:nn)'; dis= min(abs(i-loc),nn-abs(i-loc));
     pat(:,loc)=exp(-(dis*dx).^2/(2*sig^2));
 end
 w=pat*pat'; w=w/w(1,1); w=4*(w-C);
%% Update with localised input
 tall = []; rall = [];
 I_ext=zeros(nn,1); I_ext(nn/2-floor(nn/10):nn/2+floor(nn/10))=1;
 [t,u]=ode45('rnn_ode',[0 10],zeros(1,nn),[],nn,dx,w,I_ext);
 r=1./(1+exp(-u)); tall=[tall;t]; rall=[rall;r];
%% Update without input
 I_ext=zeros(nn,1);
 [t,u]=ode45('rnn_ode',[10 20],u(size(u,1),:),[],nn,dx,w,I_ext);
 r=1./(1+exp(-u)); tall=[tall;t]; rall=[rall;r];
%% Plotting results
 surf(tall',1:nn,rall','linestyle','none'); view(0,90);
```

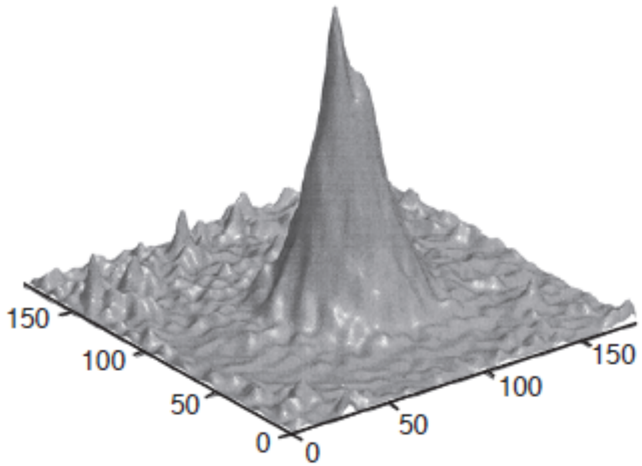
# rnn.ode

```
function udot=rnn_ode(t,u,flag,nn,dx,w,I_ext)
% odefile for recurrent network
 tau_inv = 1.;       % inverse of membrane time constant
 r=1./(1+exp(-u));
 sum=w*r*dx;
 udot=tau_inv*(-u+sum+I_ext);
return
```

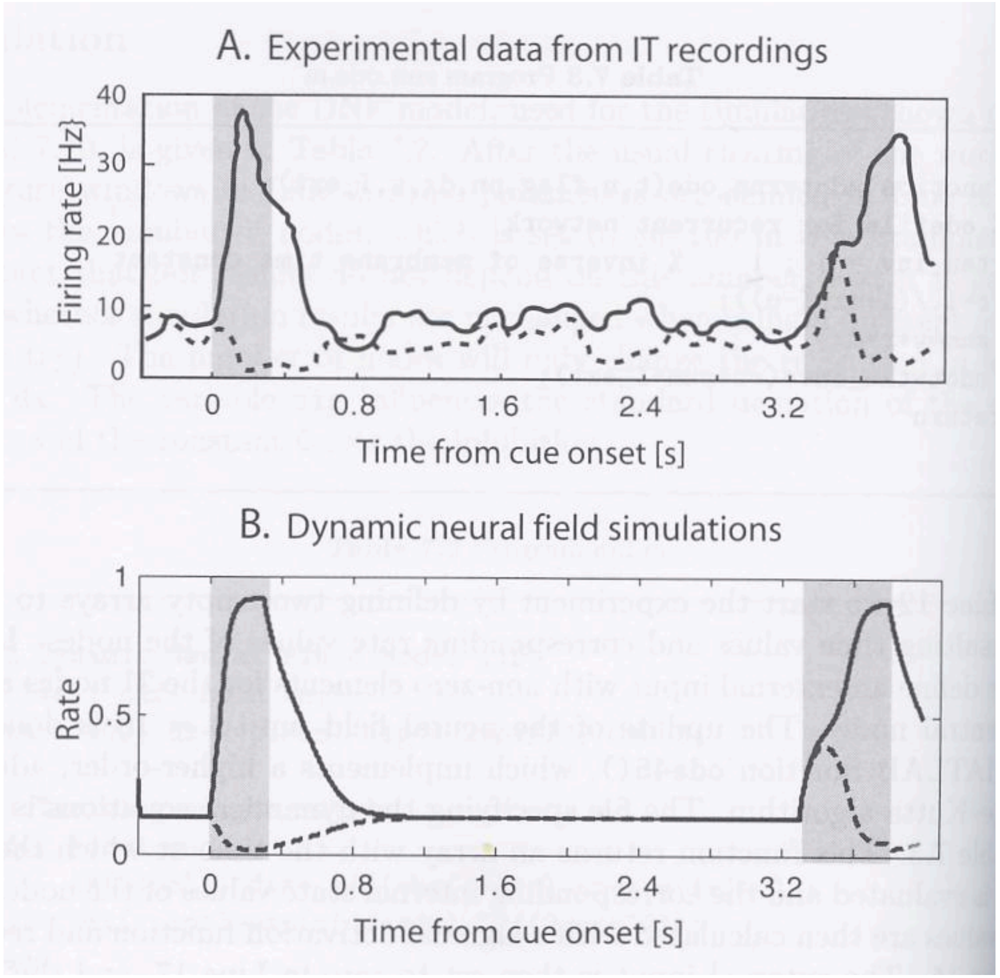
# DNF example




PFC (Funahashi, Bruce & Goldman-Rakic, 1989)




Hippocampus (Samsonovich & McNaughton, 1997)




IT (Heinke and Mavritsaki, 2009)

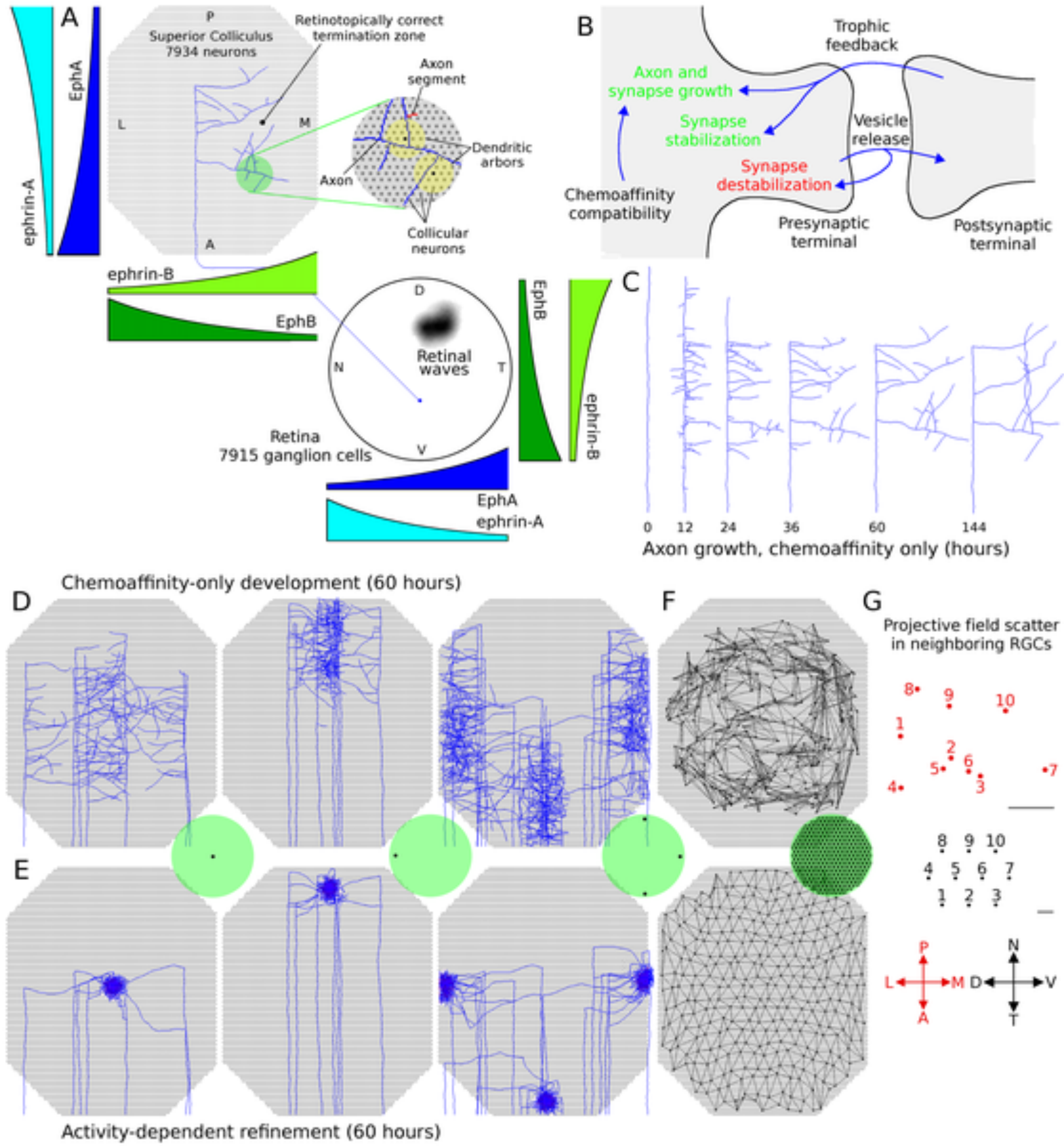
# Neural competition (lateral inhibition) everywhere?

Our results suggest

that synaptic plasticity is realized by variation in the number of synapses between neurons, not through alteration of individual synaptic weights;

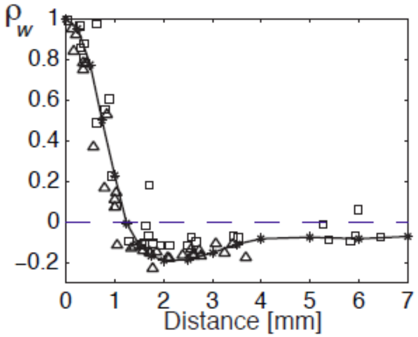
that lateral connectivity between collicular neurons is not required for organization;

and that axon arbor development does not require the gradient tracking abilities of growth cones.

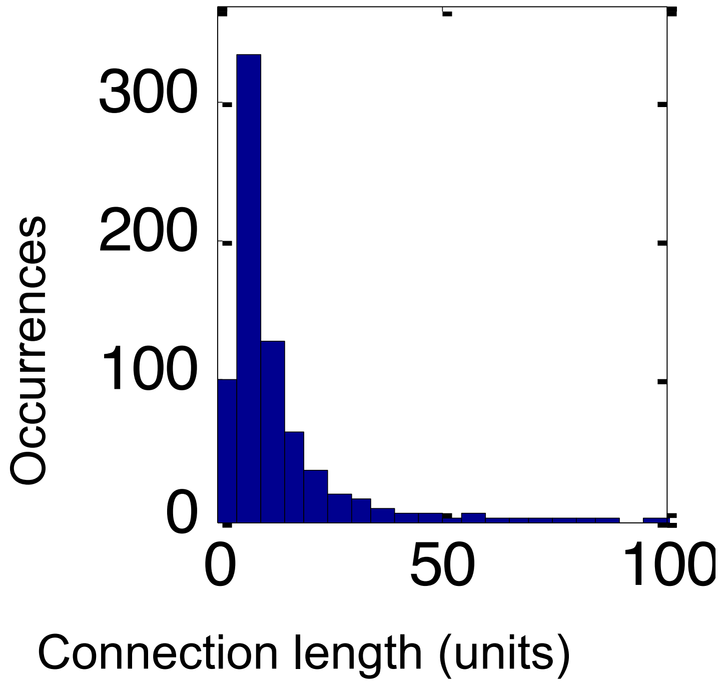


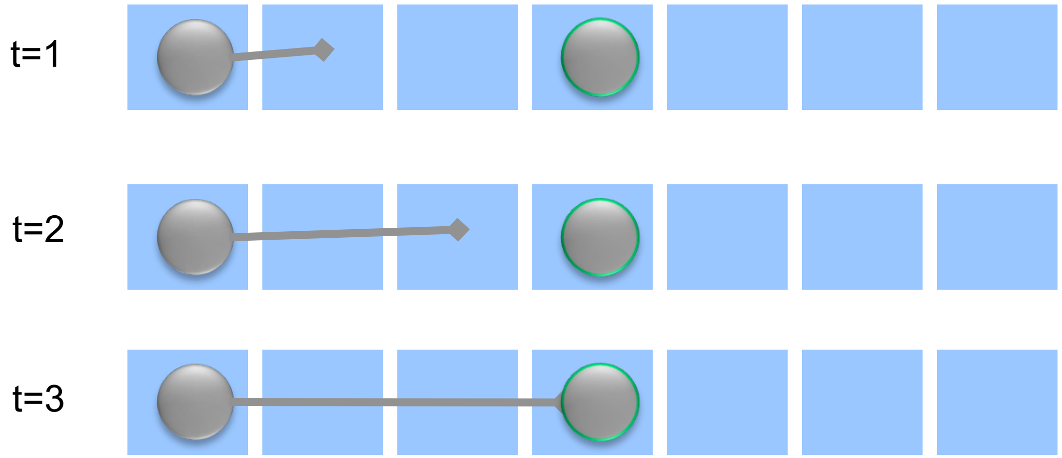

Godfrey et al. PLoS Comput Biol, 2009

# Where does the distance decay come from?




X: number of steps until another neuron
is in the range of the axonal growth cone
p: probability that unit space contains a neuron
q = 1 - p




$$P(X = n) = p * q^{n-1}$$

-> exponential distribution
-> “Mexican hat” can be explained
through random axon growth

Kaiser et al. Cerebral Cortex, 2009

# Summary

Topographic maps

Self-organizing maps

- Willshaw & von der Malsburg
- Kohonen
- Dynamic neural Field

# Further readings

Teuvo Kohonen (1989), **Self-organization and associative memory**, Springer Verlag, 3rd edition.

David J. Willshaw and Christoph von der Malsburg (1976), **How patterned neural connexions can be set up by self-organisation**, in **Proc Roy Soc** B 194, 431–445.

Shun-ichi Amari (1977), **Dynamic pattern formation in lateral-inhibition type neural fields**, in **Biological Cybernetics** 27: 77–87.

Huge R. Wilson and Jack D. Cowan (1973), **A mathematical theory of the functional dynamics of cortical and thalamic nervous tissue**, in **Kybernetik** 13:55-80.

Kechen Zhang (1996), **Representation of spatial orientation by the intrinsic dynamics of the head-direction cell ensemble: A theory**, in **Journal of Neuroscience** 16: 2112–2126.

Simon M. Stringer, Thomas P. Trappenberg, Edmund T. Rolls, and Ivan E.T. de Araujo (2002), **Self-organizing continuous attractor networks and path integration I: One-dimensional models of head direction cells**, in **Network: Computation in Neural Systems** 13:217–242.