Dynamical Systems

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Lecture 1 of 5

Announcement:

- All slides (I keep updating them), exercises, coursework:

- **https://tinyurl.com/jpabcoz**

Overview

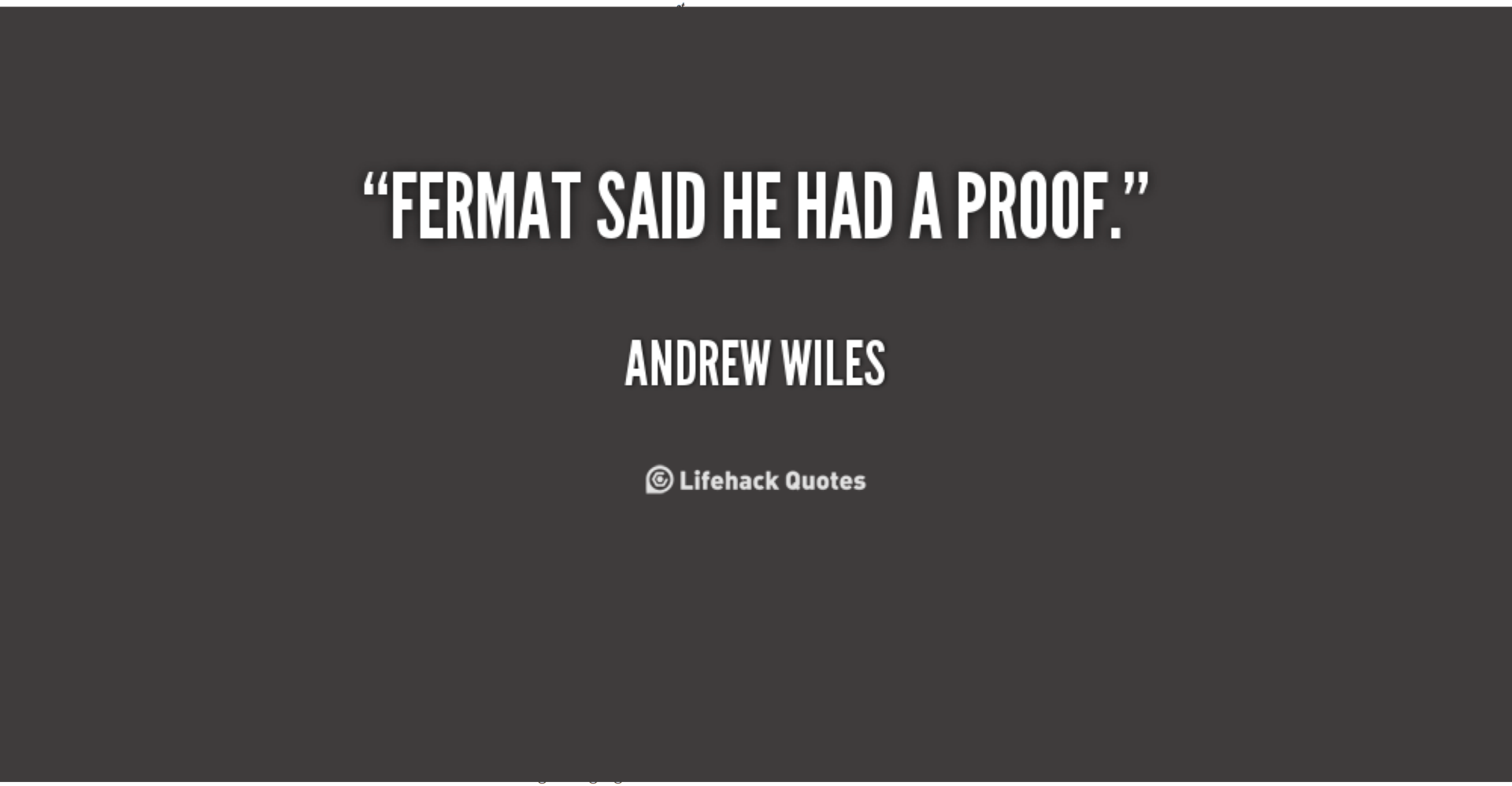
- What are dynamical systems?
- How to interpret a differential equation
- How to analyse differential equation systems
- How to solve differential equation systems
- Stability analysis, multistability
- Oscillatory solutions
- Parameter variations, bifurcations
- Choice of cool stuff: Chaos, turbulence, spatio-temporal systems, slow-fast systems, transients, and more.

A lot of things to take in…

so ask immediately if anything is unclear.

Disclaimer

mit einer stetigen Funktion $a(t) \geq 0$ und einer nichtfallenden Funktion $b(t) \geq 0$, so folgt



bilitätssatzes mit $g(t,x) = f(t,x)$ und $v_0 = u_0$. Die Stabilitätsabschätzung ergibt dann notwendig $u(t) = v(t)$ für alle $t \in I$. Q.E.D.

Also…

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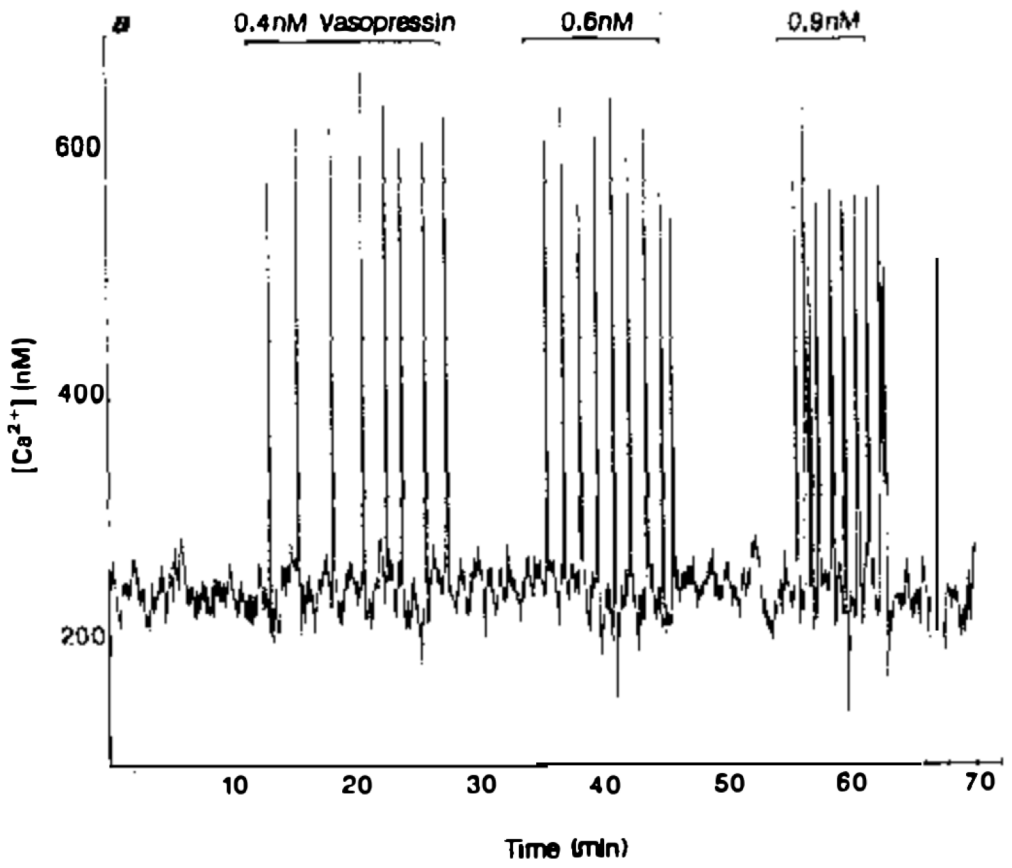
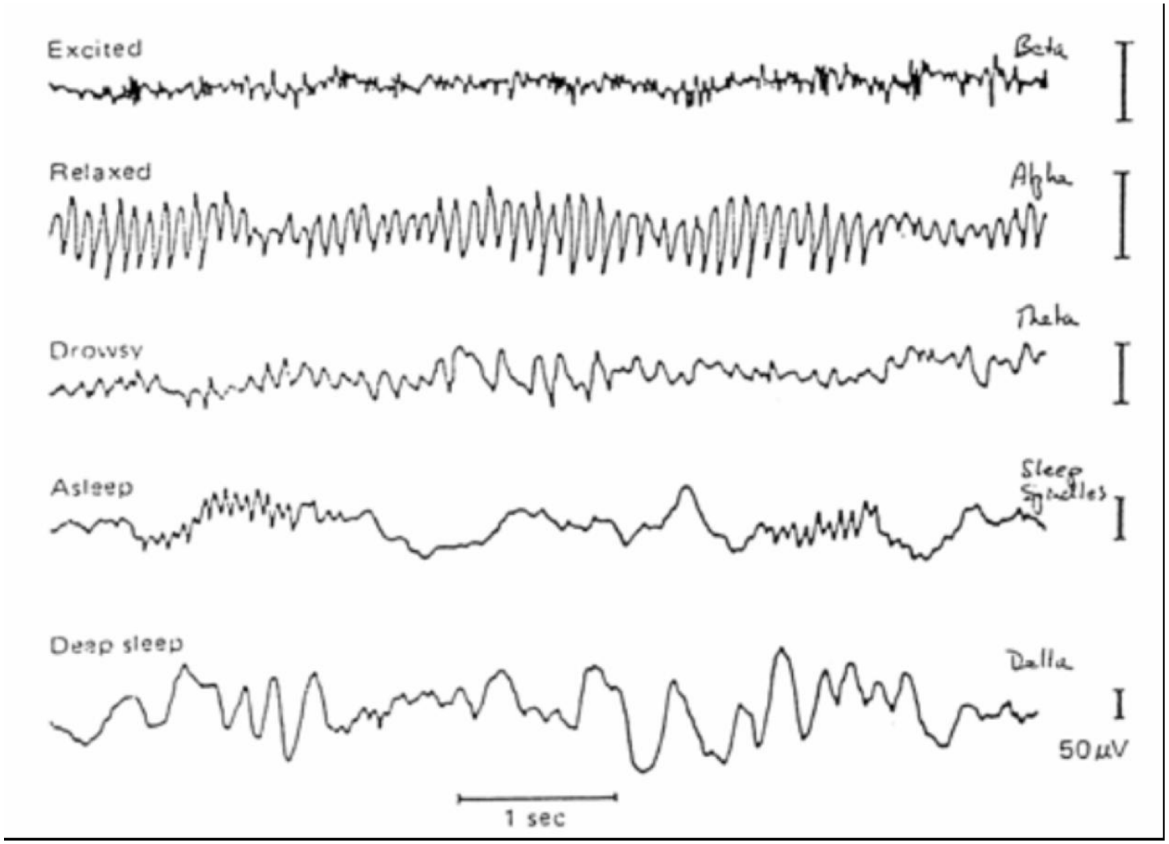
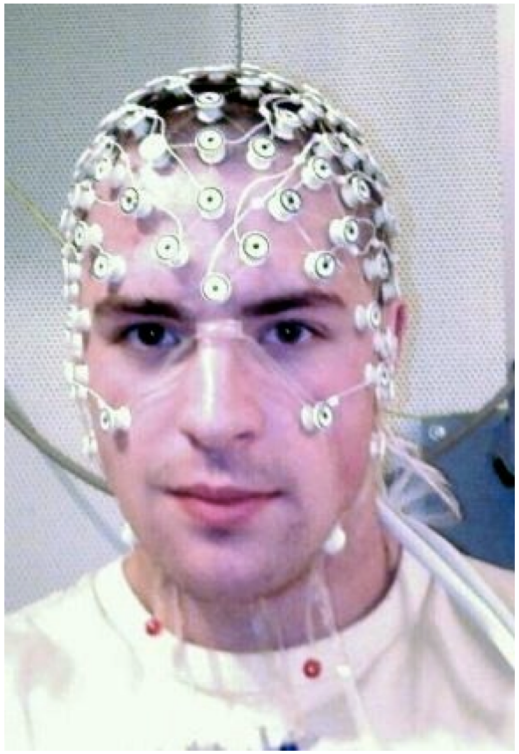
Example: Calcium spiking



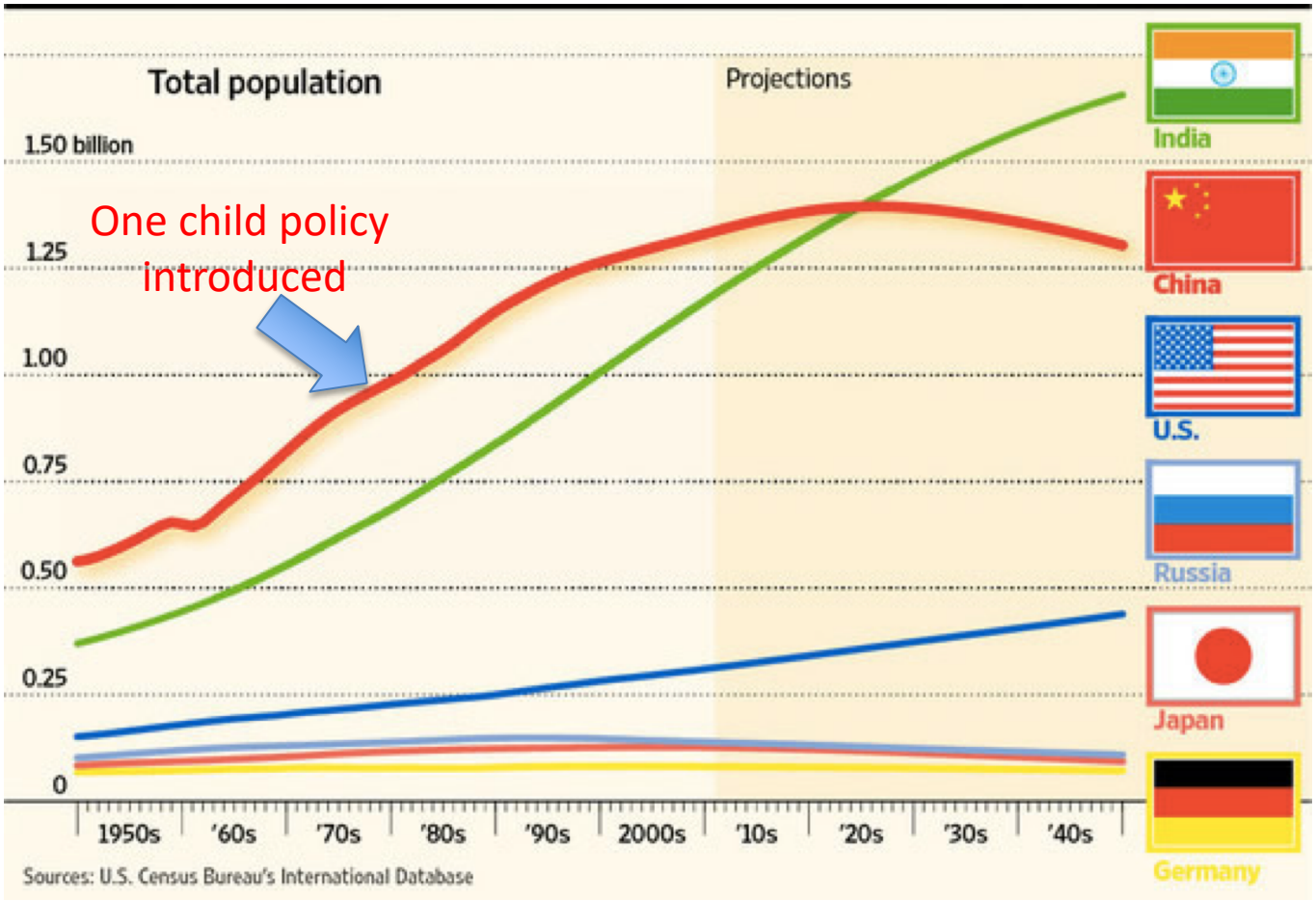
Example: Electroencephalographic recordings (EEG)



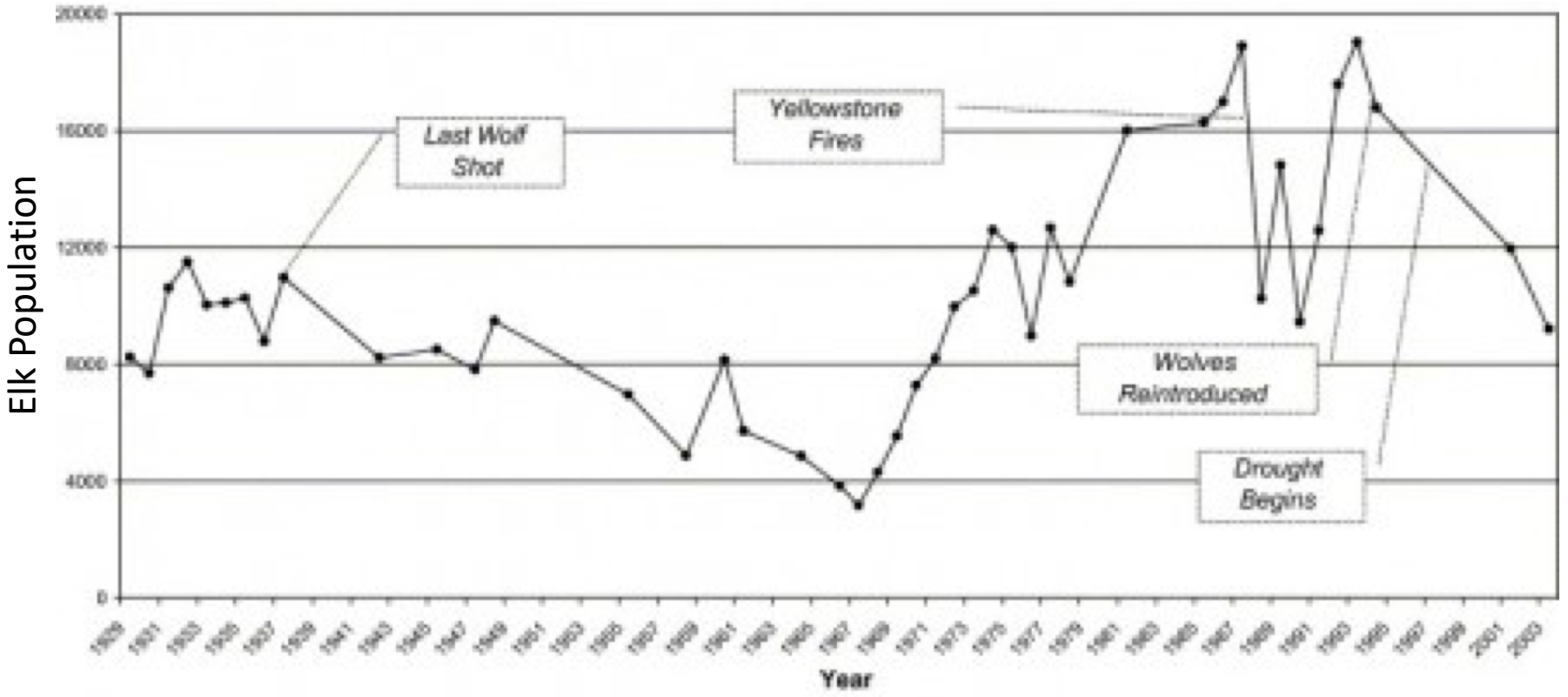
- Electrical potential on the scalp



Example: Population Growth



Example: Elk population in Yellowstone



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Describing dynamical systems: Discrete systems

$$x(n+1) = x(n) + 1$$

$$x(0) = 0$$

Describing dynamical systems: Discrete systems

$$x(n+1) = x(n) + c$$

$$x(0) = a$$

Describing dynamical systems: Continuous systems

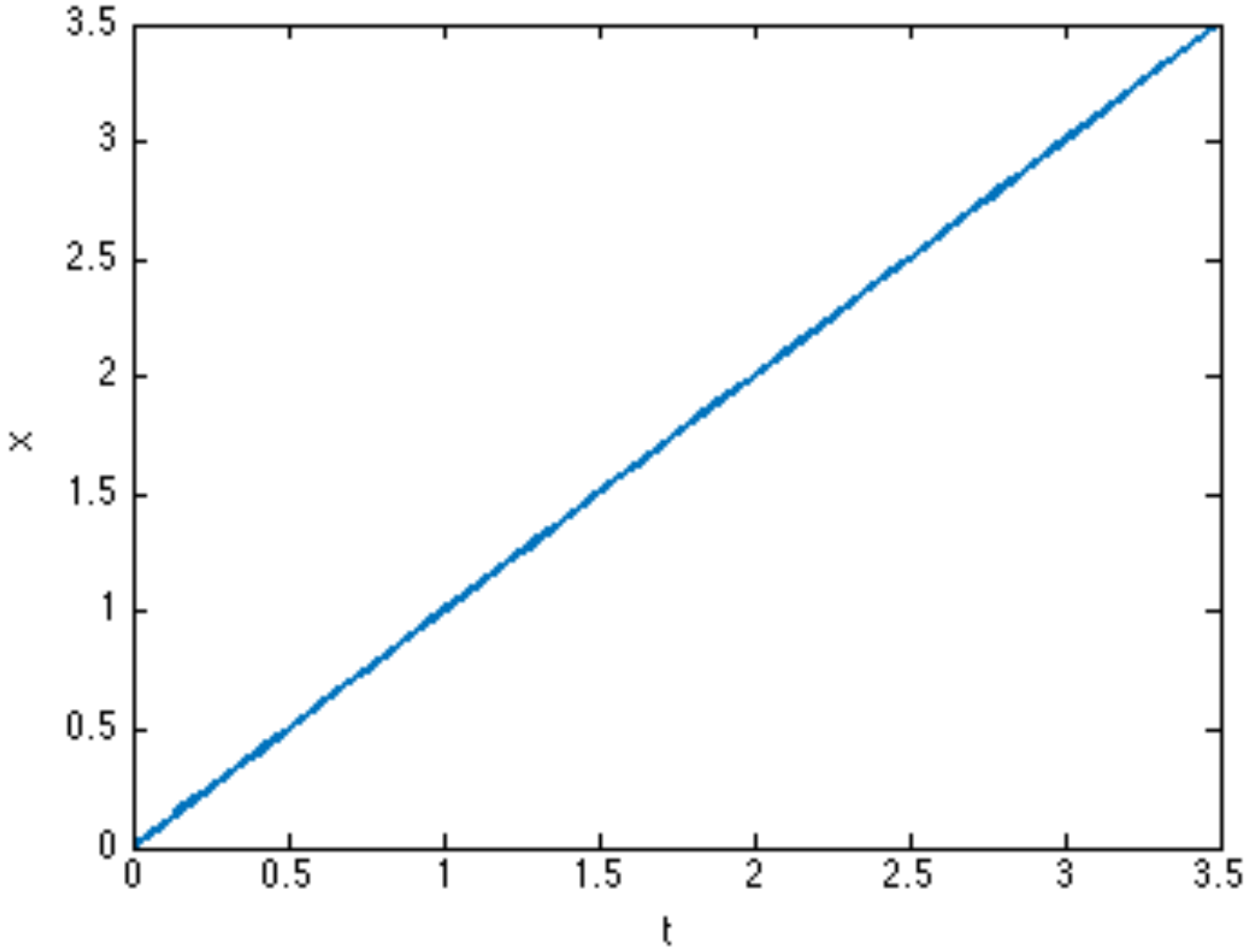
$$x(n+1) = x(n) + 1$$

…

make continuous

…

$$dx/dt=1$$

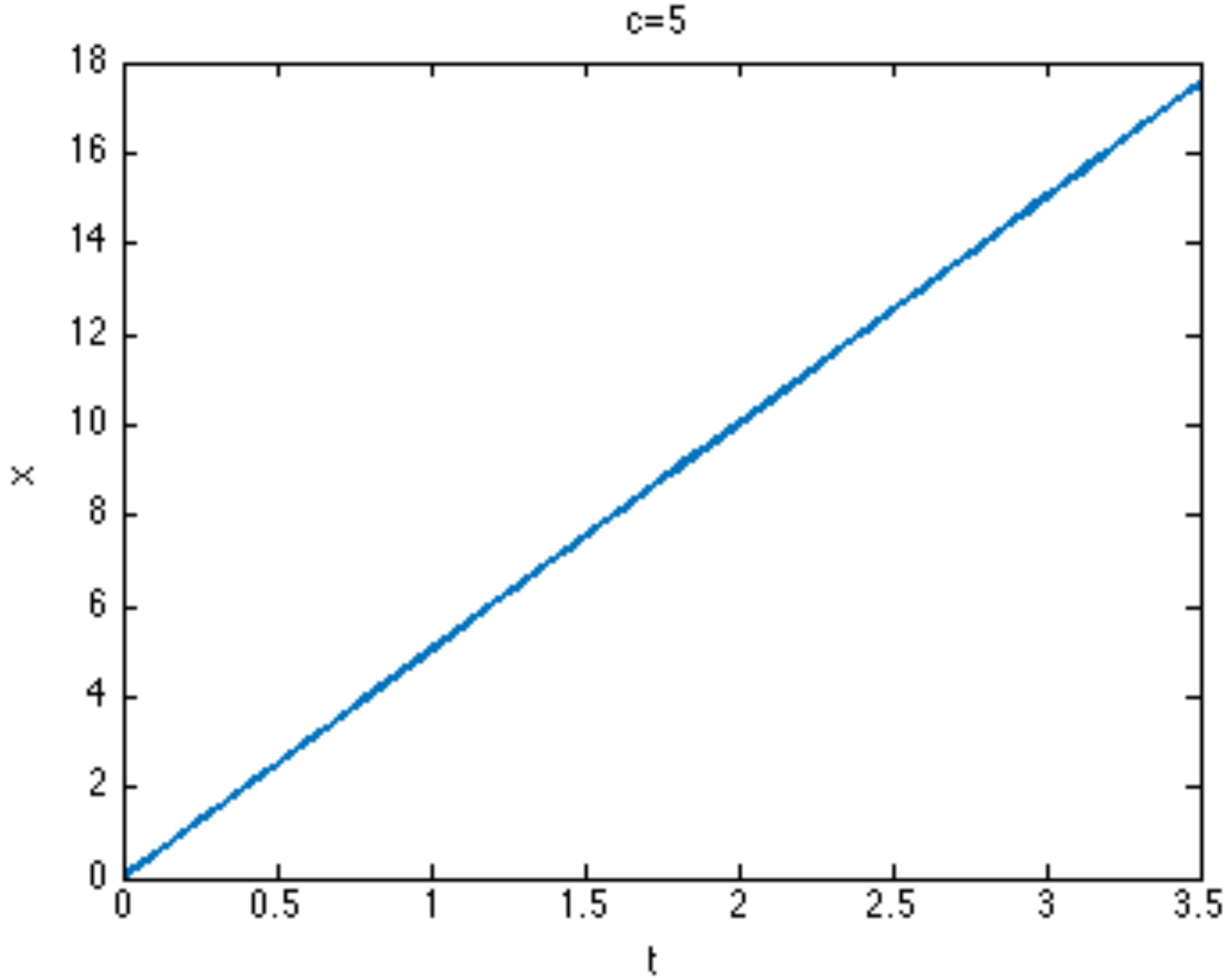

3.5
3
2.5
2
1.5
1
0.5
0
x
0
0.5
1
1.5
2
2.5
3
3.5
t

Describing dynamical systems: Continuous systems

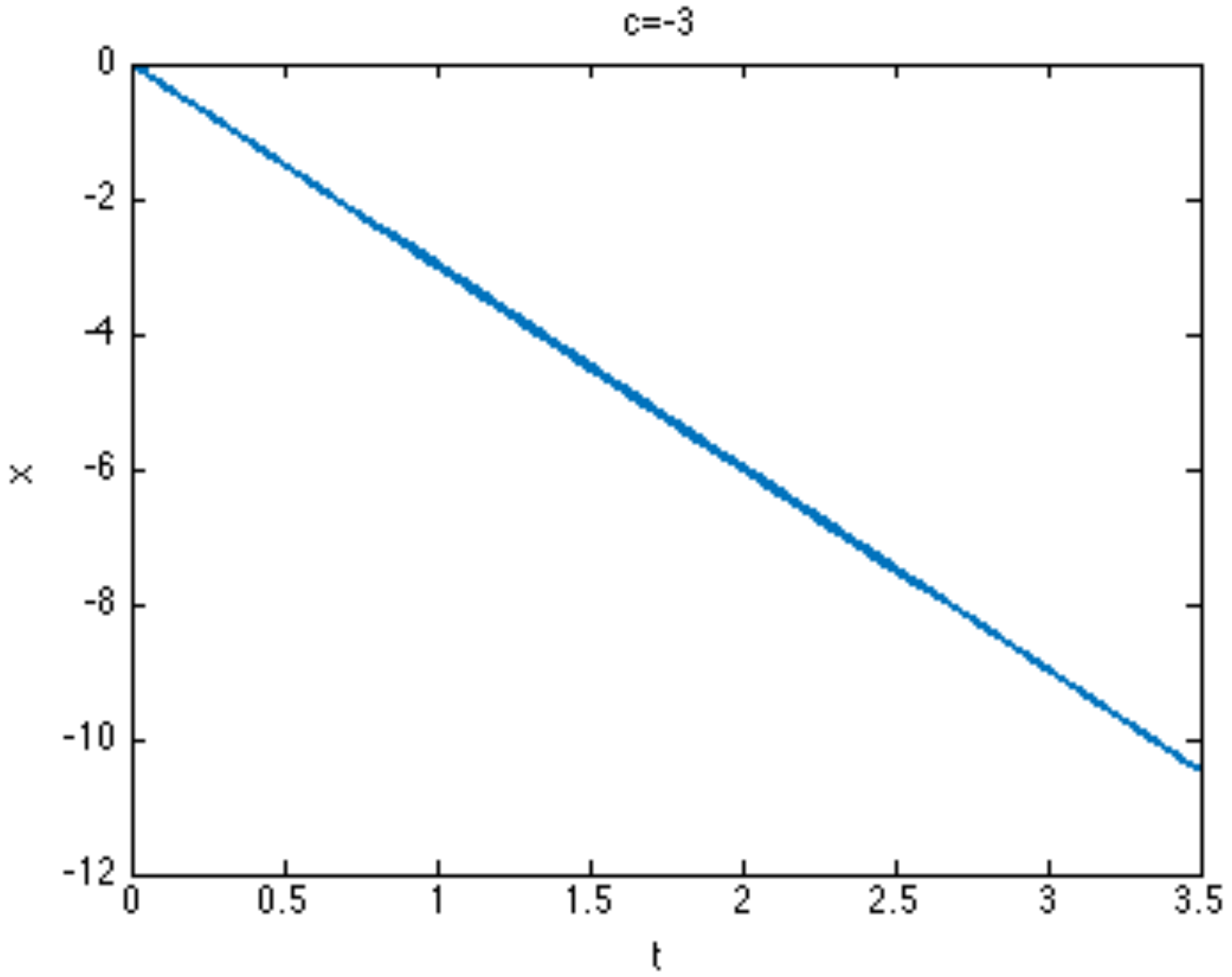
$$x(n+1) = x(n) + c$$

$$dx/dt=c$$

C=5, x(0)=0



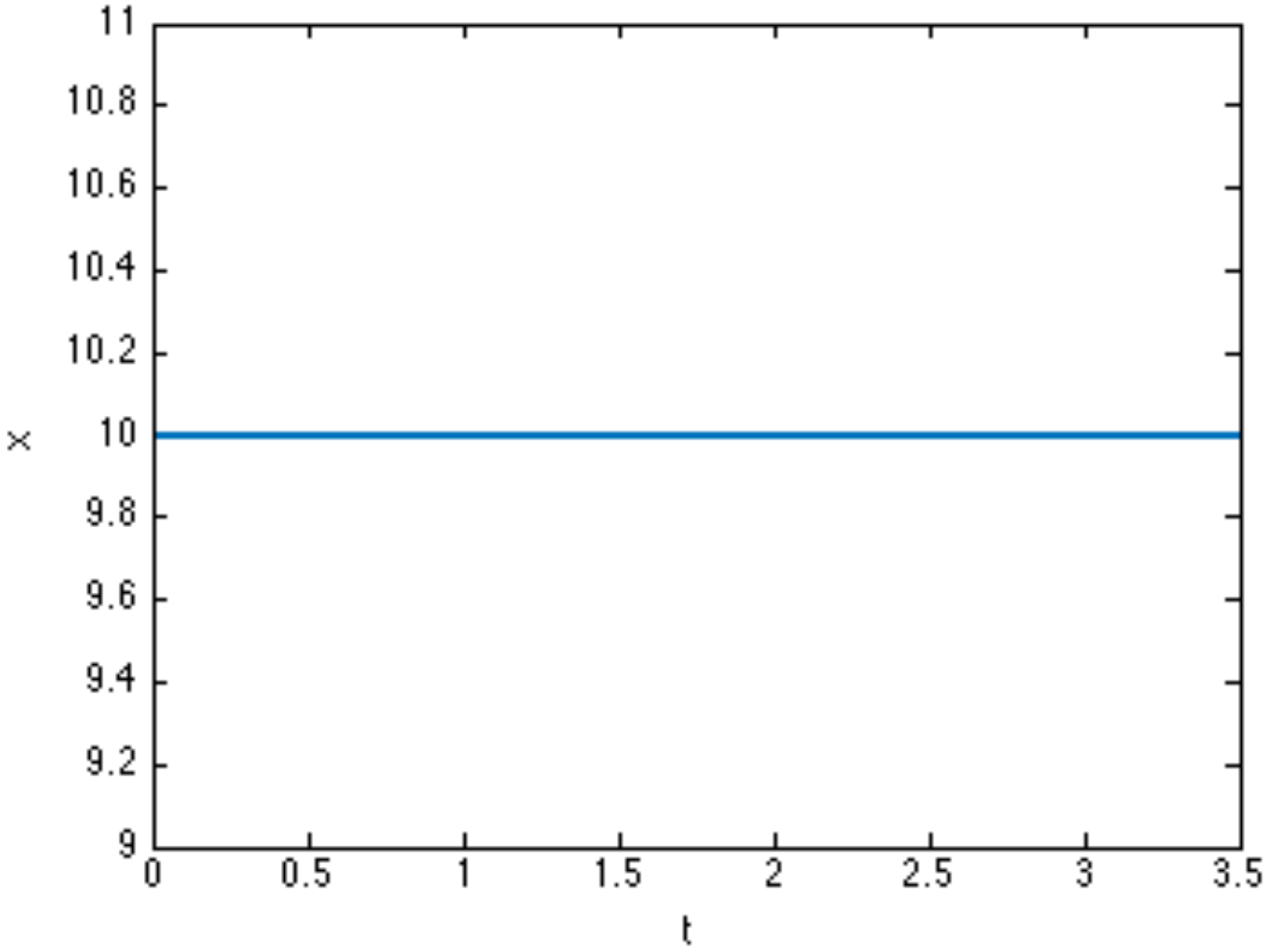
C=-3, x(0)=0



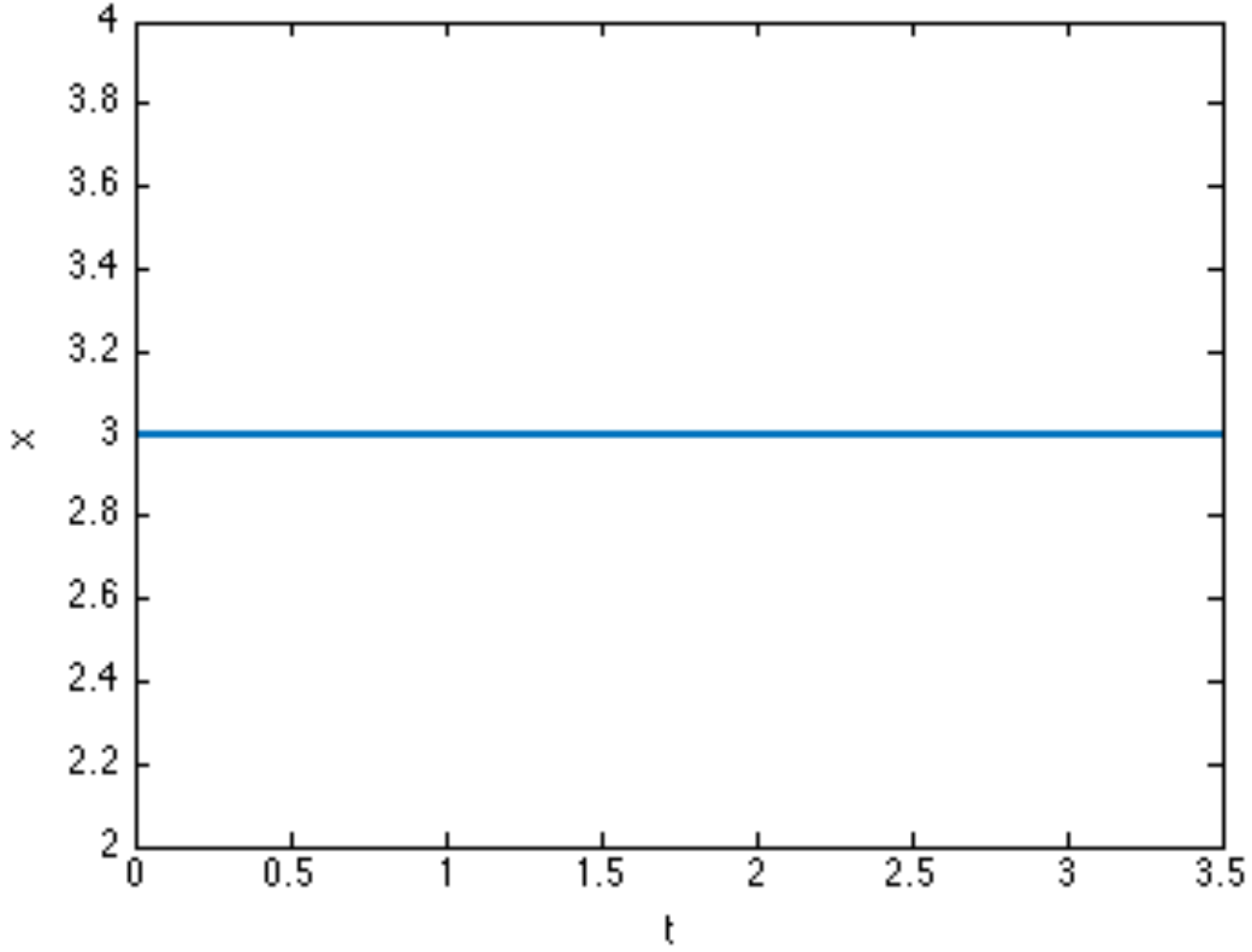
Describing dynamical systems: Continuous system

$$dx/dt=0$$

$$x(0)=10$$


11
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
9
x
0
0.5
1
1.5
2
2.5
3
3.5
t

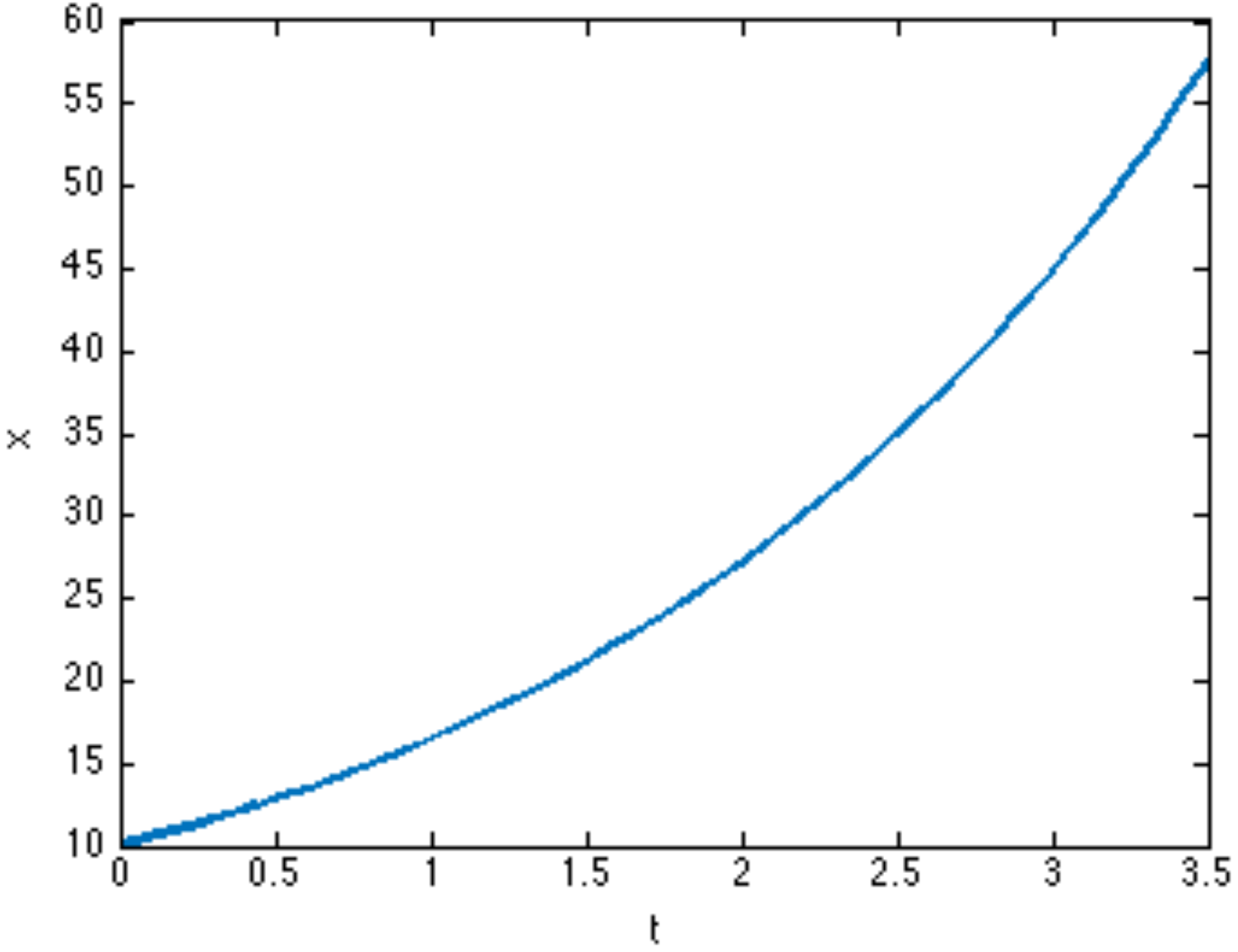
x(0)=3



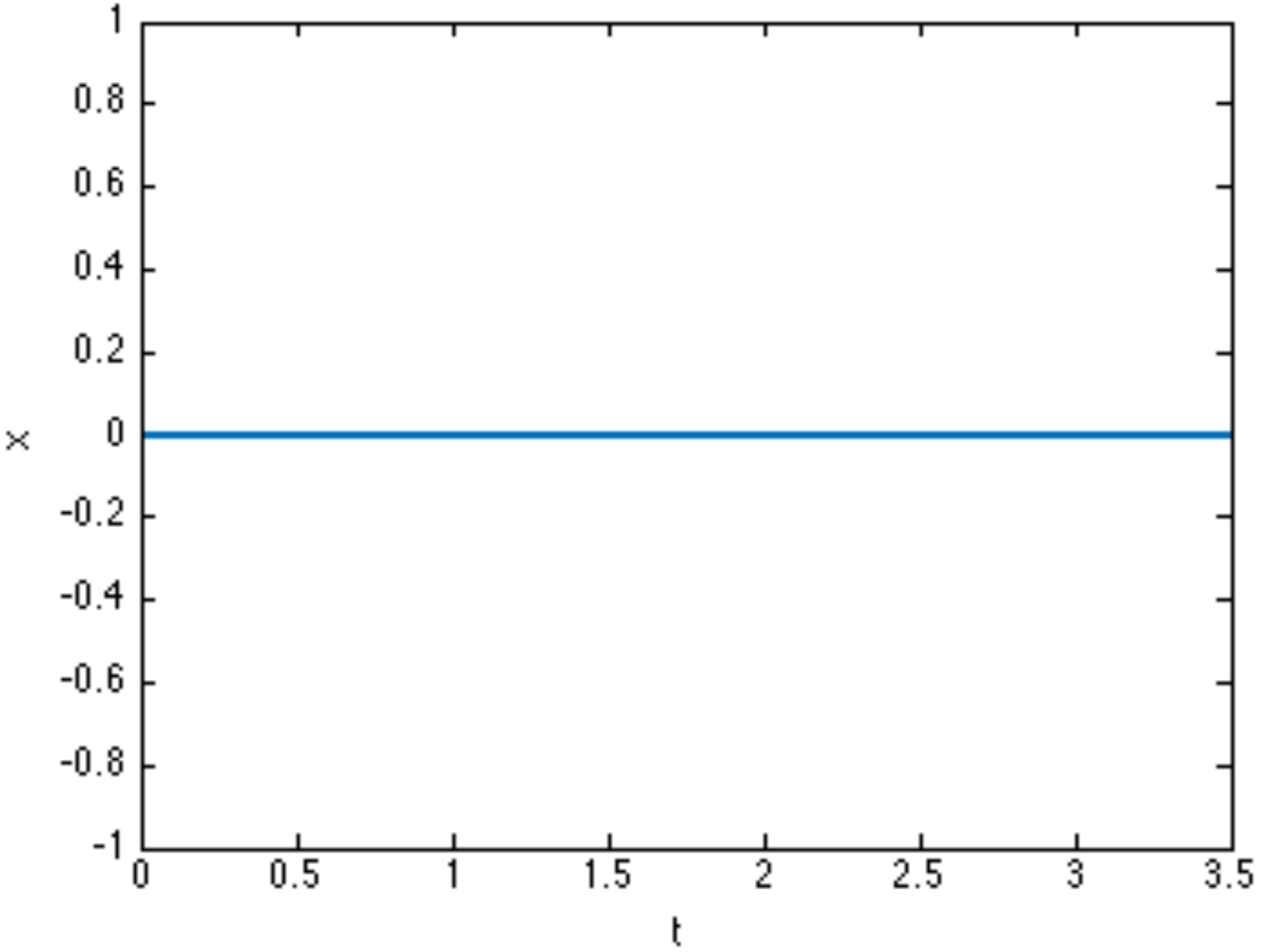
Describing dynamical systems: Continuous system

$$dx/dt=0.5x$$

$$x(0)=10$$


60
55
50
45
40
35
30
25
20
15
10
x
0
0.5
1
1.5
2
2.5
3
3.5
t

x(0)=0 ?

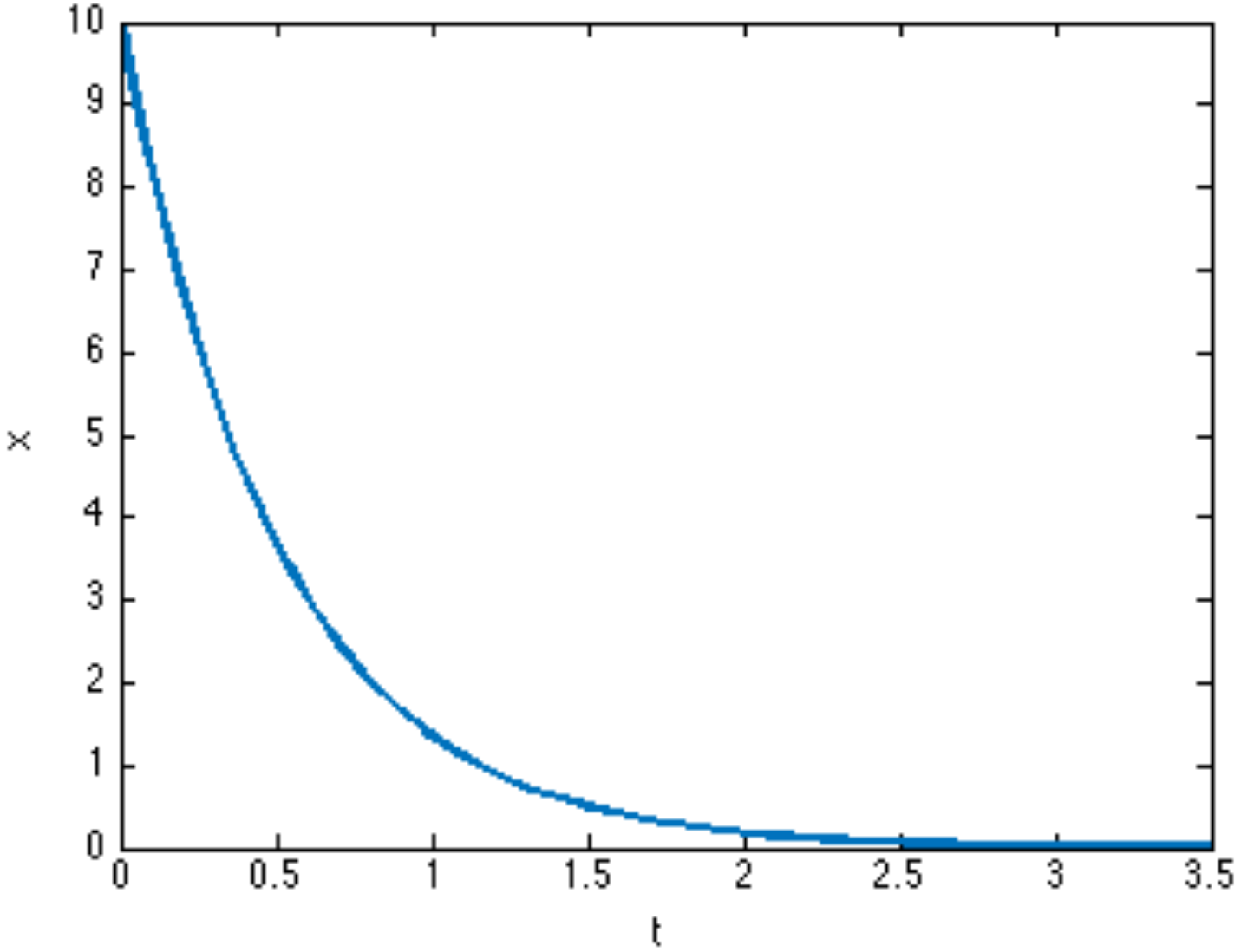


Describing dynamical systems: Continuous systems

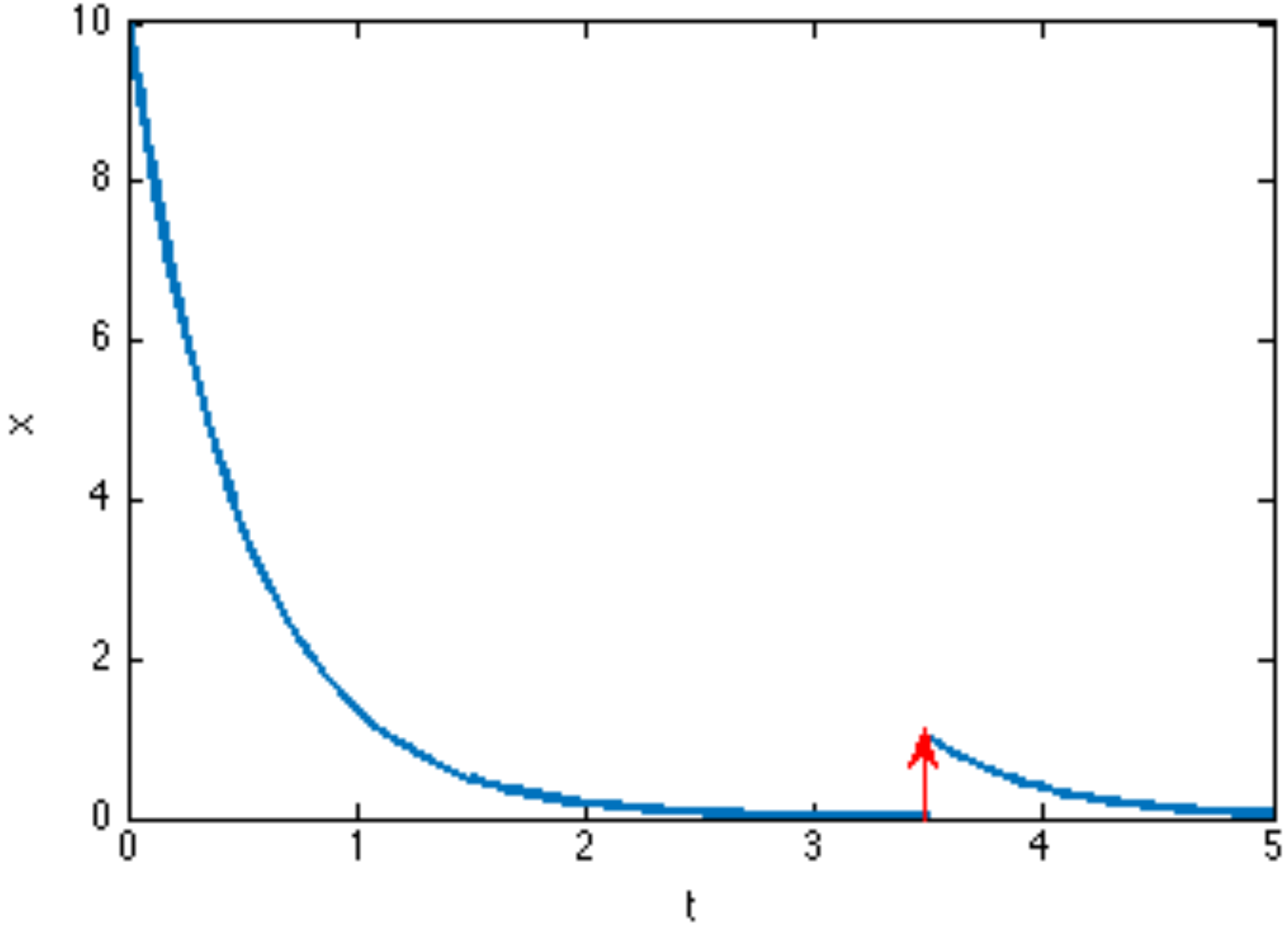
$$dx/dt = k*x$$

$$x(0) = 10$$

k<0



Perturbation



Terminology recap

- Variable or state
- Differential equation
- Initial condition
- Trajectory
- Parameter
- Steady state
- Transient behaviour
- Perturbation

Enzyme catalysed reaction

[ADP] -> ADP concentration

[ATP] -> ATP concentration

[ATPS] -> ATP synthase concentration

$$\frac{d[ADP]}{dt} = -r*[ATPS]*[ADP]$$

$$\frac{d[ATP]}{dt} = +r*[ATPS]*[ADP]$$

$$\frac{d[ATPS]}{dt} = c - k*[ATPS]$$

r -> ATP conversion rate

c -> ATPS transcription/translation rate

k -> ATPS degradation rate

Feedback inhibition

[X] -> Protein X concentration
[Y] -> Protein Y is the dimer of XX

$$d[X]/dt = -k*X - r/2*X*X + (g-Y)$$
$$d[Y]/dt = r*X*X - l*Y$$

k: degredation rate of X, l: degradation rate of Y
r: dimerisation rate
g: production rate of X

Enzyme conversion: Michaelis Menten kinetics

[S] -> Substrate concentration

[P] -> Product concentration

$$d[S]/dt = -d[P]/dt$$

$$d[P]/dt = (Vmax*[S])/(Km+[S])$$

Vmax: maximum conversion speed

Km: concentration of [S] at which the conversion speed is half of Vmax

A remark on terminology

- $dx/dt=f(x)$ is called an ordinary differential equation.
- $dx/dt=f(x,y)$, $dy/dt=g(x,y)$ is called a 2 dimensional ordinary differential equation system.
- $dx1/dt=f(x1,x2,x3,\ldots xn)$, $dx2/dt=g(x1,x2,x3,\ldots,xn)$ … is called a n dimensional ordinary differential equation system.